600 Mechanik der Kontinua

610 *Feste Körper*
620 *Flüssigkeiten und Gase*

um was geht es?

Beschreibung von Bewegungen (phys. Verhalten) des nicht-starren Körpers (elastisch, plastisch) → Kontinuum

Hydro- und Aerodynamik

Kompartimentale Modellierung

Wellen und Wellenphänomene

Strömung um einen Modellhubschrauber

621 Druck in Flüssigkeiten und Gasen

621 Ziele

- Druckverläufe in Wasser und Atmosphäre (barometrische Höhenformel) berechnen können
- Auftriebskräfte berechnen können

621 Theorie

Druck

$$p = \frac{dF_N}{dA}$$

$$\left[p\right] = N/m^2 = Pa$$

621 Theorie

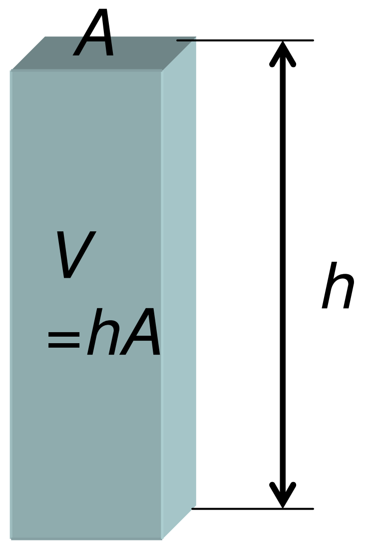
Druck

$$p = \frac{dF_N}{dA}$$

$$[p] = N/m^2 = Pa$$

Schweredruck,
Hydrostatischer Druck

$$F_G = mg = \rho V g$$



621 Theorie

Druck

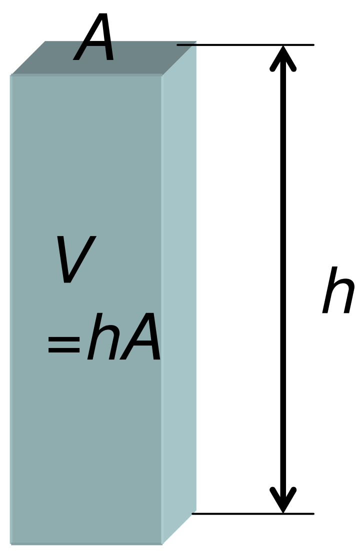
$$p = \frac{dF_N}{dA}$$

$$[p] = N / m^2 = Pa$$

Schweredruck,
Hydrostatischer Druck

$$F_G = mg = \rho V g$$

$$p = \frac{F_G}{A}$$



621 Theorie

Druck

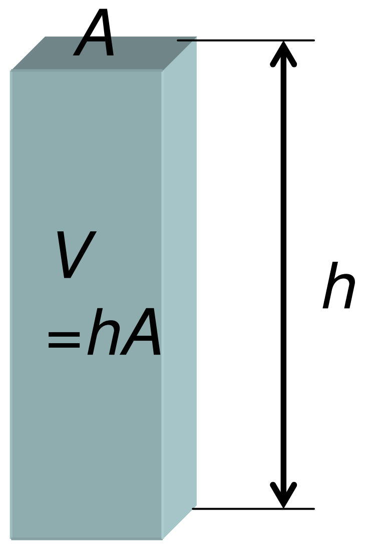
$$p = \frac{dF_N}{dA}$$

$$[p] = N/m^2 = Pa$$

Schweredruck,
Hydrostatischer Druck

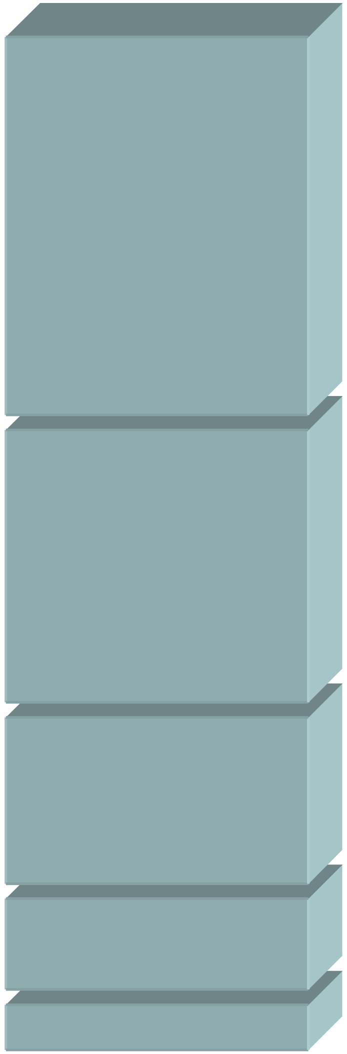
$$F_G = mg = \rho V g$$

$$p = \frac{\rho g V}{A} = \rho g h$$



621 Theorie

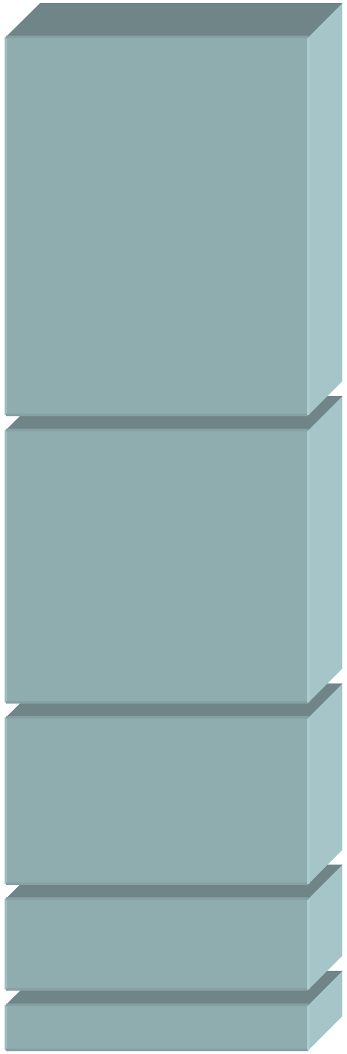
Atmosphäre: Was ist anders als im Wasser?



621 Theorie

Atmosphäre: Was ist anders als im Wasser?

→ Kompressibilität



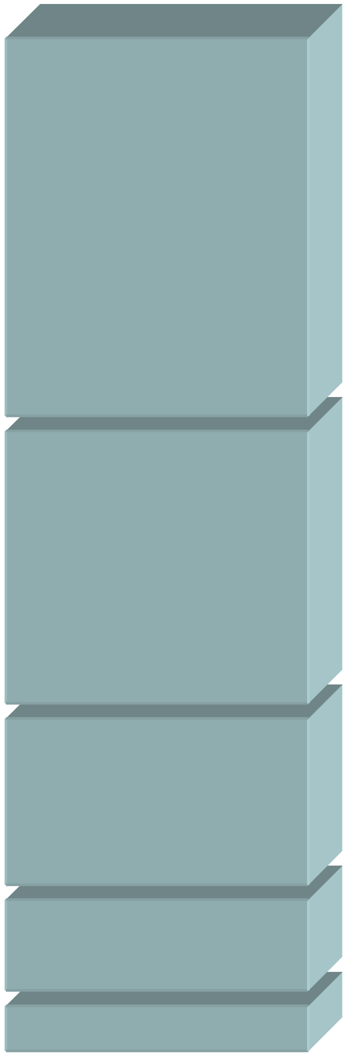
621 Theorie

Atmosphäre: Was ist anders als im Wasser?

→ Kompressibilität

$$dp = -\rho(z) \cdot g \cdot dz$$

621 Theorie



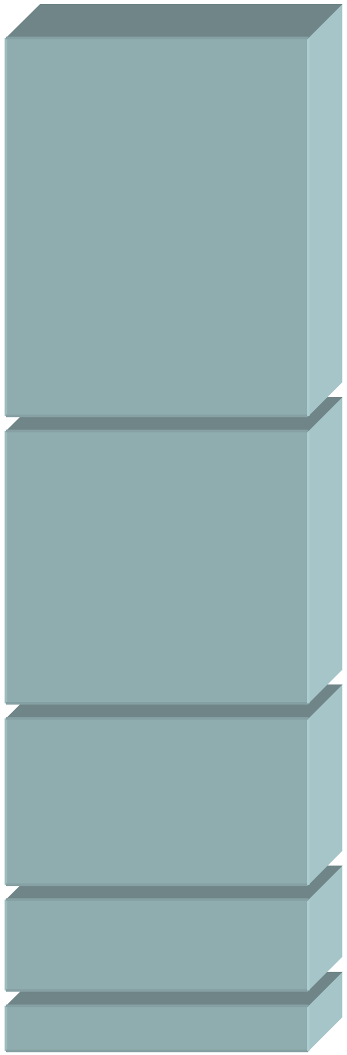
Atmosphäre: Was ist anders als im Wasser?

→ Kompressibilität

$$dp = -\rho(z) \cdot g \cdot dz$$

$$\frac{\rho(z)}{\rho_0} = \frac{p(z)}{p_0}$$

(Idelaes Gas → 700)

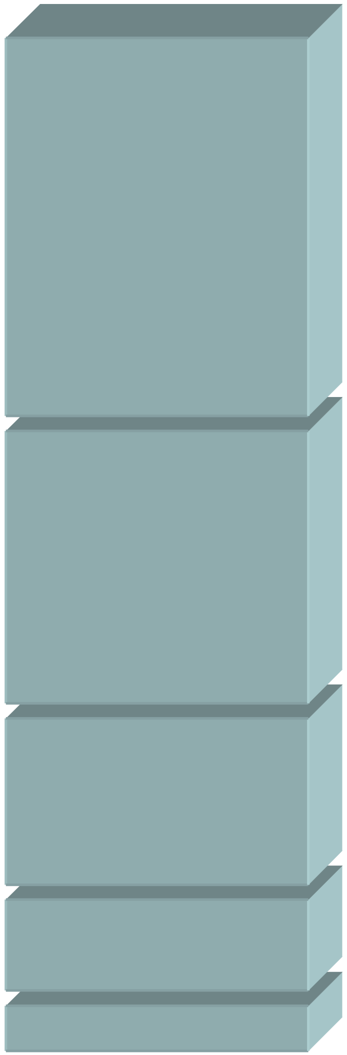


621 Theorie

$$\frac{\rho(z)}{\rho_0} = \frac{p(z)}{p_0}$$

$$dp = -\rho(z) \cdot g \cdot dz$$

$$\boxed{\frac{dp}{dz} = -\frac{\rho_0 \cdot g}{p_0} \cdot p}$$

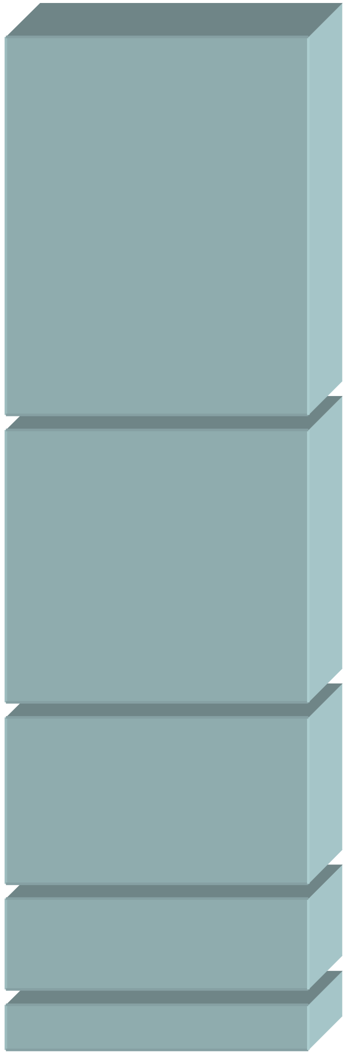


621 Theorie

Lösen durch Separation und Integration

$$\frac{dp}{dz} = -\frac{\rho_0 \cdot g}{p_0} \cdot p$$

$$\int \frac{1}{p} dp = -\frac{\rho_0 g}{p_0} \int dz$$



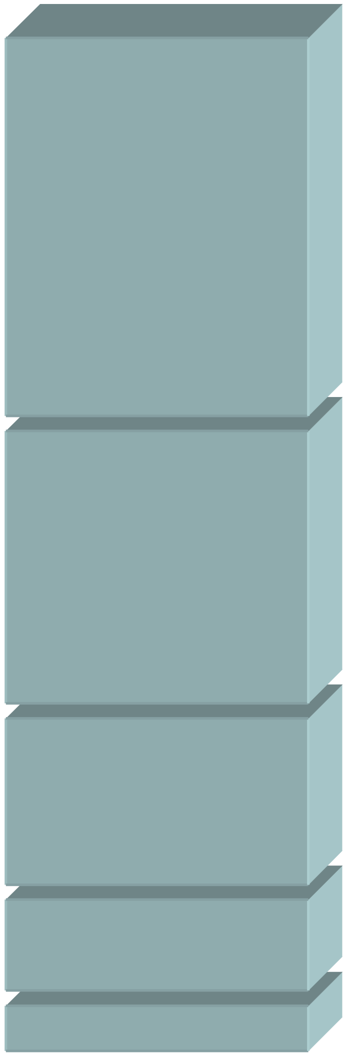
621 Theorie

Lösen durch Separation und Integration

$$\frac{dp}{dz} = -\frac{\rho_0 \cdot g}{p_0} \cdot p$$

$$\int \frac{1}{p} dp = -\frac{\rho_0 g}{p_0} \int dz$$

$$\ln p = -\frac{\rho_0 g}{p_0} z + c$$



621 Theorie

Lösen durch Separation und Integration

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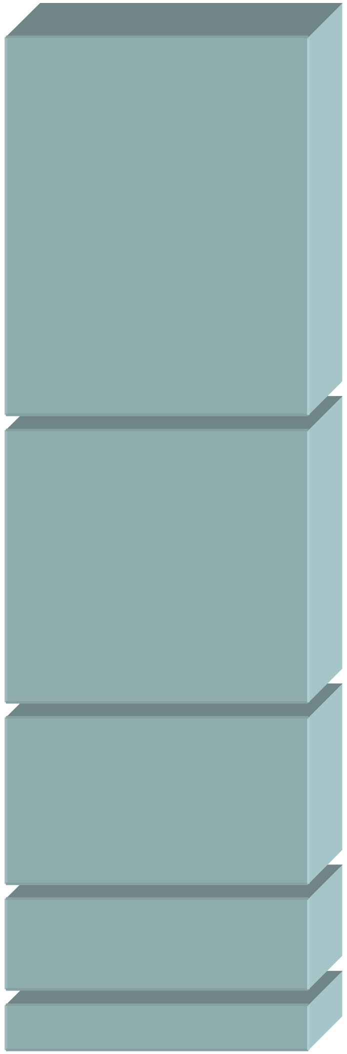
$$\int \frac{1}{p} dp = -\frac{\rho_0 g}{p_0} \int dz$$

$$\ln p = -\frac{\rho_0 g}{p_0} z + c$$

$$p(z) = e^{-\frac{\rho_0 g}{p_0} \cdot z} \cdot e^{c}$$



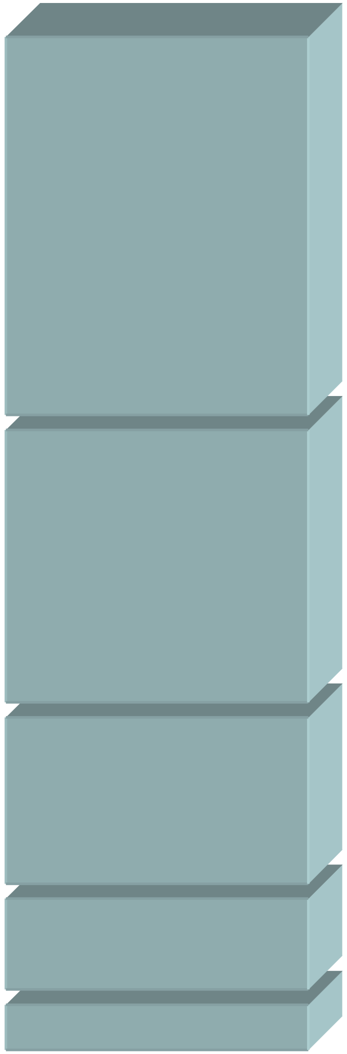
621 Theorie

Bestimmen der Konstante

$$p(z) = e^{-\frac{\rho_0 g}{p_0}\cdot z} \cdot e^c$$

$$p(0) = e^c = p_0$$

$$\boxed{p(z) = p_0 \cdot e^{-\frac{\rho_0 g}{p_0}\cdot z}}$$

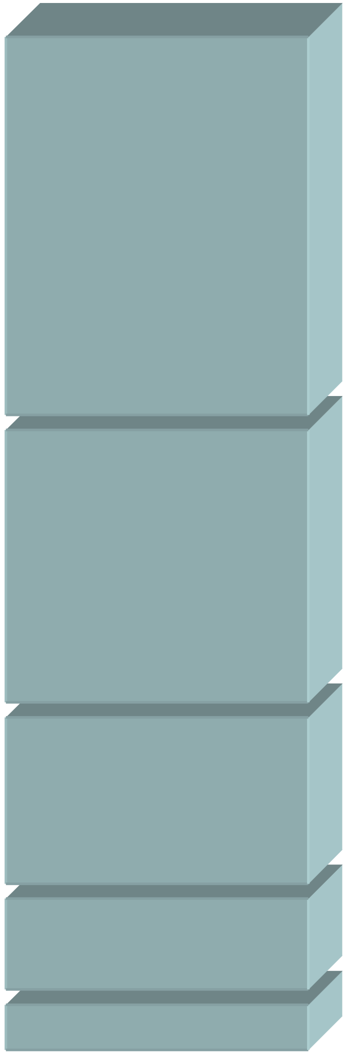


621 Theorie

Höhe mit halbem Druck

$$p(z) = p_0 \cdot e^{-\frac{\rho_0 g}{p_0} \cdot z}$$

$$\frac{p(z_{1/2})}{p_0} = \frac{1}{2} = e^{-\frac{\rho_0 g}{p_0} \cdot z_{1/2}}$$



621 Theorie

Höhe mit halbem Druck

$$p(z) = p_0 \cdot e^{-\frac{\rho_0 g}{p_0} \cdot z}$$

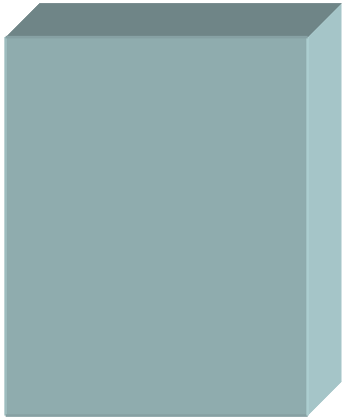
$$\frac{p(z_{1/2})}{p_0} = \frac{1}{2} = e^{-\frac{\rho_0 g}{p_0} \cdot z_{1/2}}$$

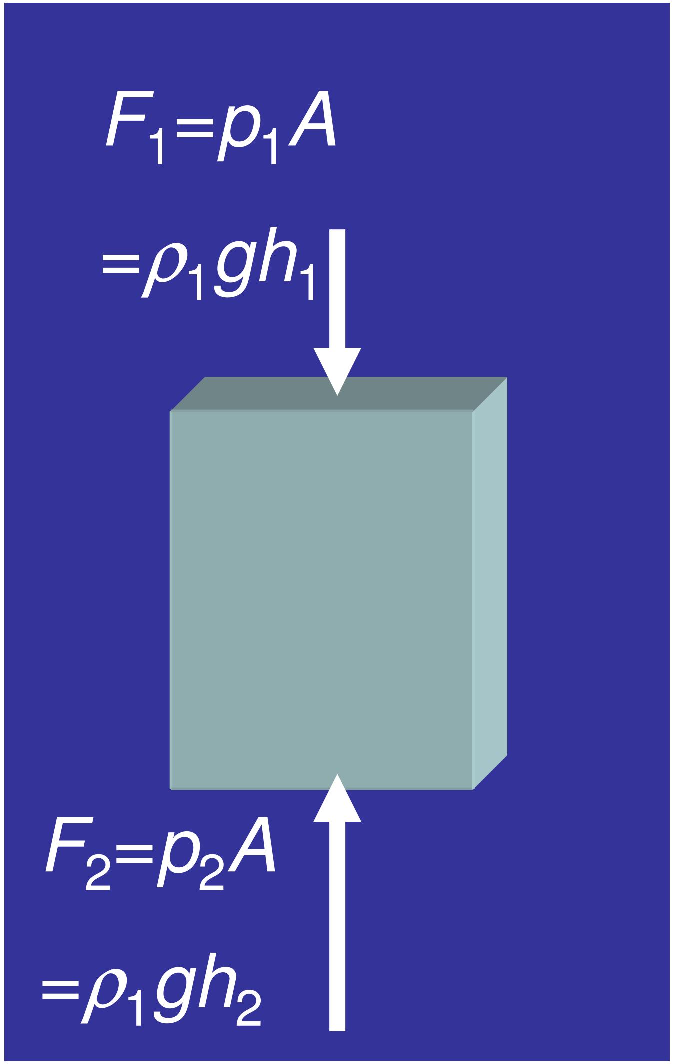
$$\boxed{z_{1/2} = -\frac{p_0 \cdot \ln(0.5)}{\rho_0 \cdot g} = \frac{p_0 \cdot \ln(2)}{\rho_0 \cdot g}}$$

621 Theorie

Vom Schwimmen und Fliegen

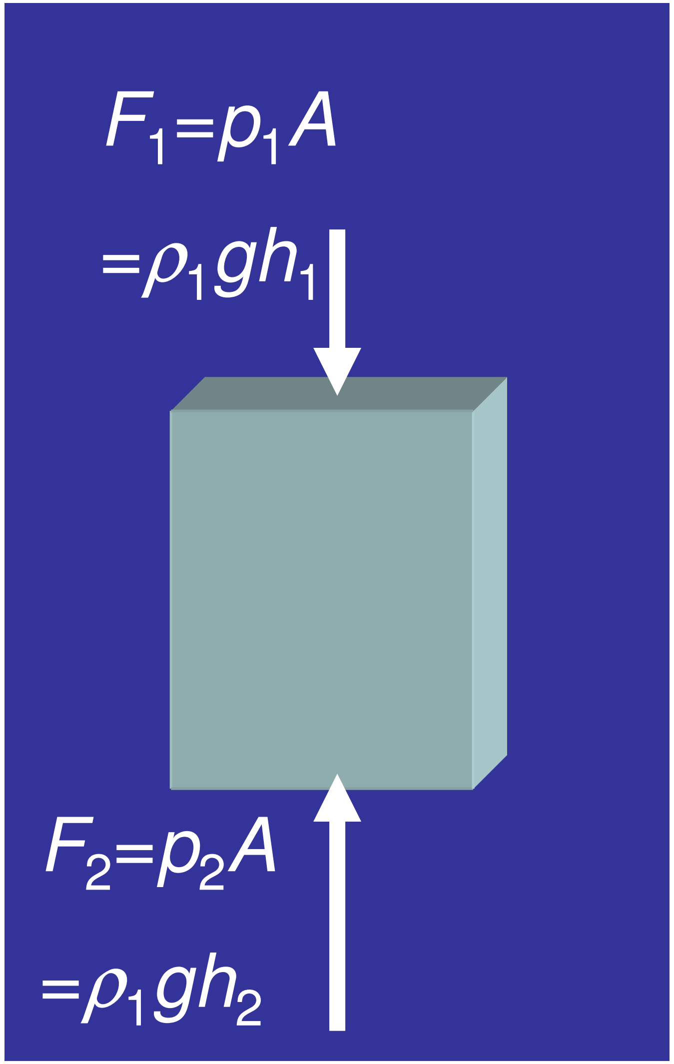
…





621 Theorie

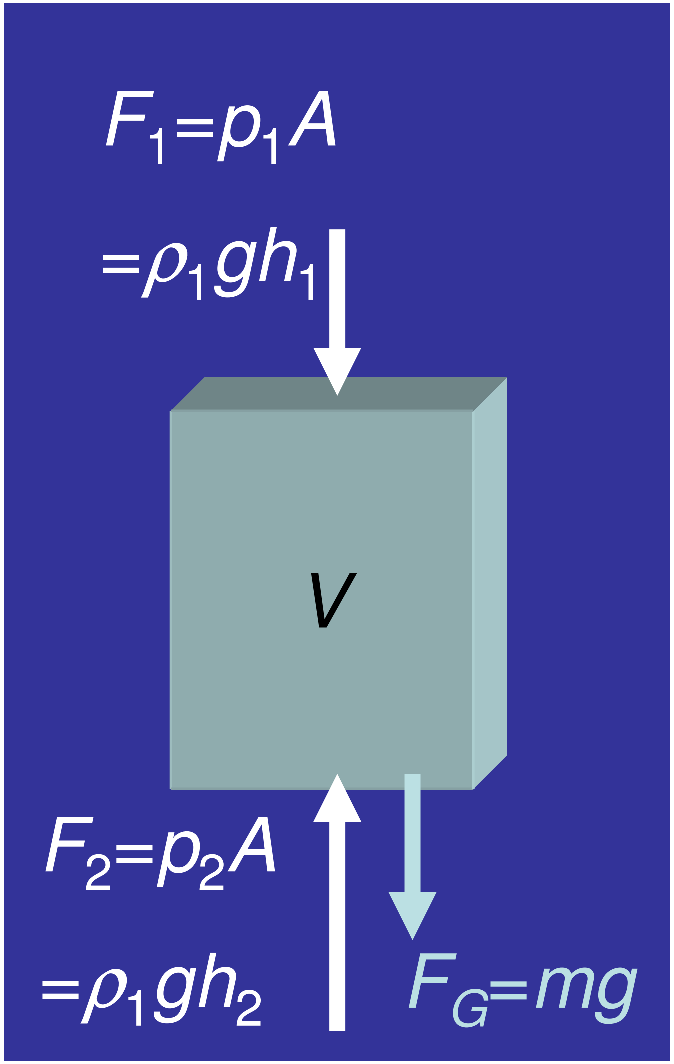
Vom Schwimmen und (Ballon-) Fliegen …



621 Theorie

Vom Schwimmen und (Ballon-) Fliegen …

$$F_A = A \cdot \Delta p$$
$$= \rho_1 g \cdot A \cdot \Delta h$$
$$= \rho_1 g \cdot V$$

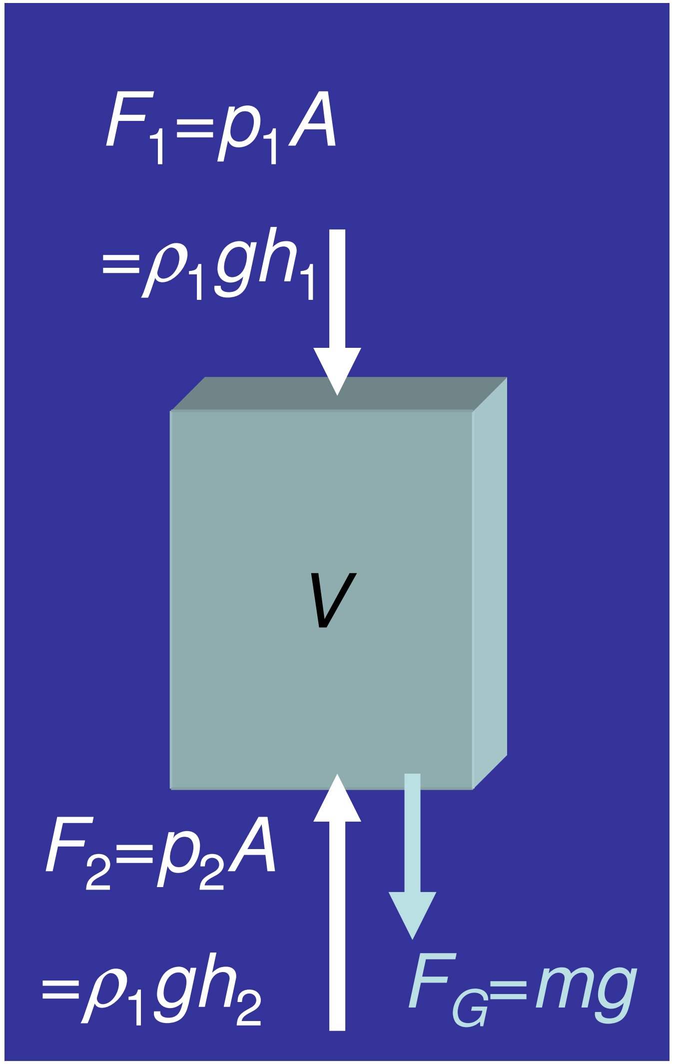


621 Theorie

Vom Schwimmen und (Ballon-) Fliegen …

$$F_A = \rho_1 g \cdot V$$

$$F = ma = F_A - F_G$$



621 Theorie

Vom Schwimmen und (Ballon-) Fliegen …

$$F_A = \rho_1 g \cdot V$$

$$F = ma = F_A - F_G$$

$$ma = \rho_1 g V - mg$$

$$= \rho_1 g V - \rho_2 g V$$

$$= \Delta\rho \cdot g V$$

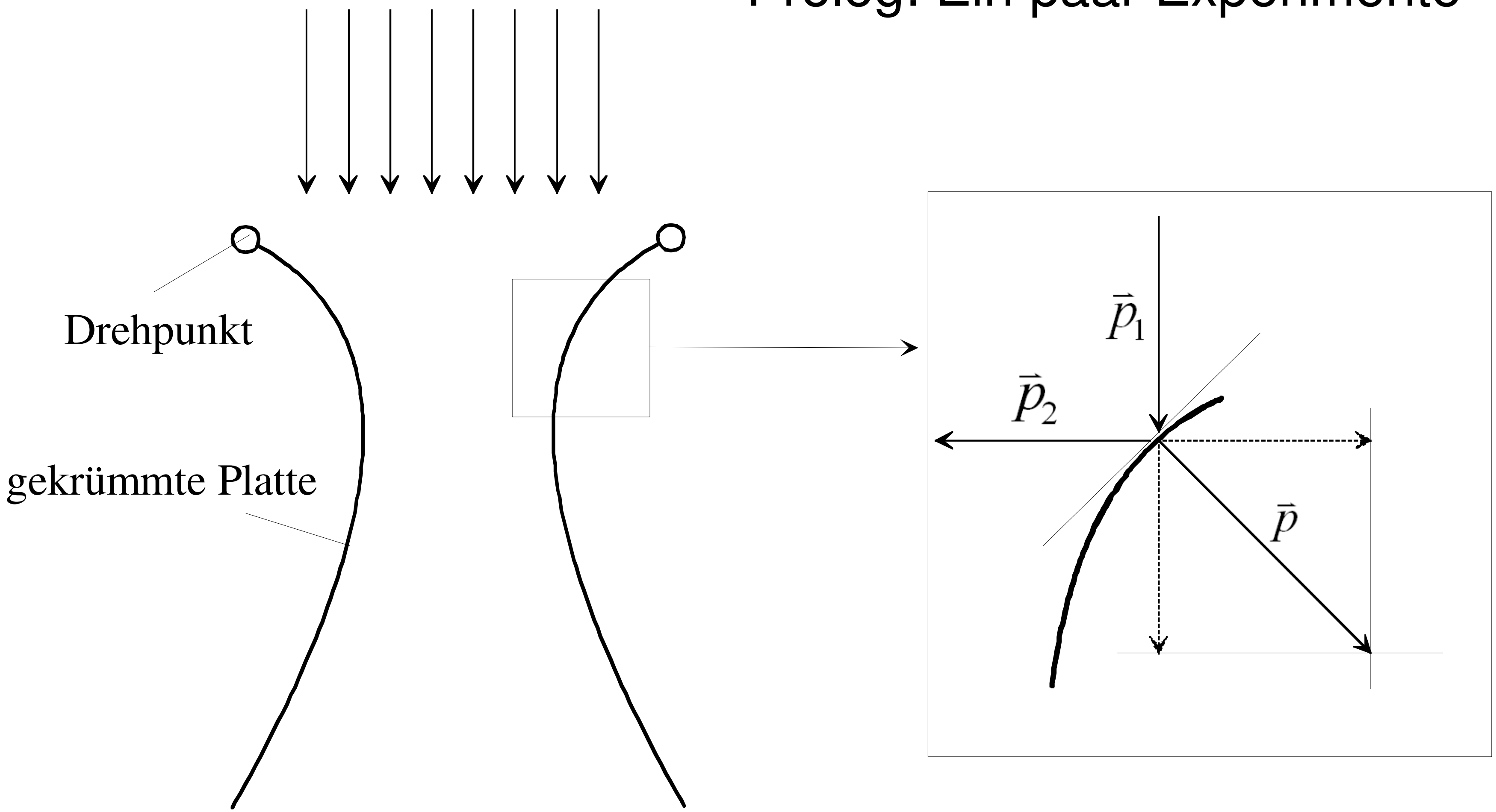
622 stationäre, ideale Strömungen

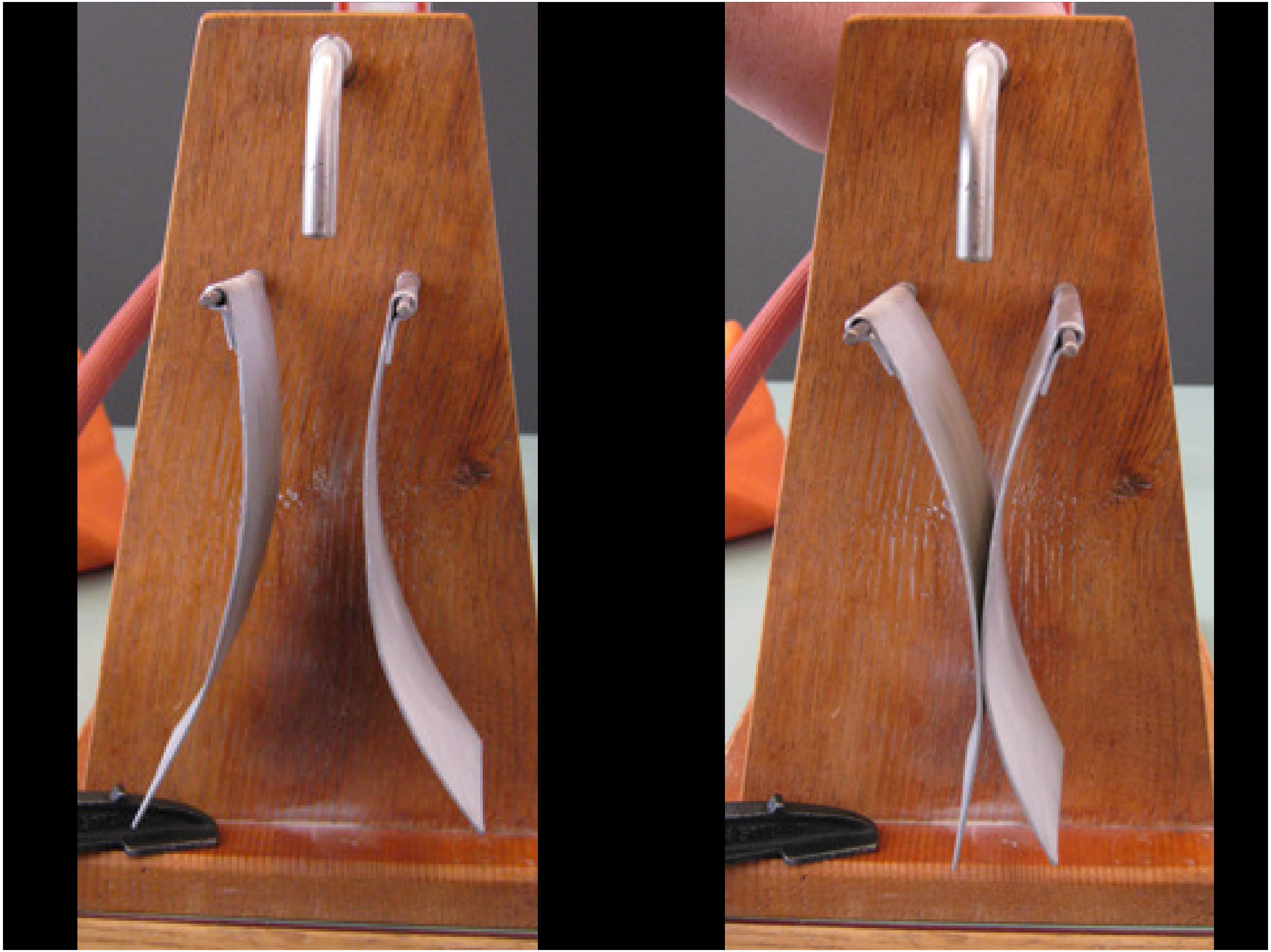
622 Ziele

- Drücke in einer idealen, laminaren Strömung berechnen können
- Erklären können, warum sich in einer Strömung Druckänderungen ergeben können

622 Theorie

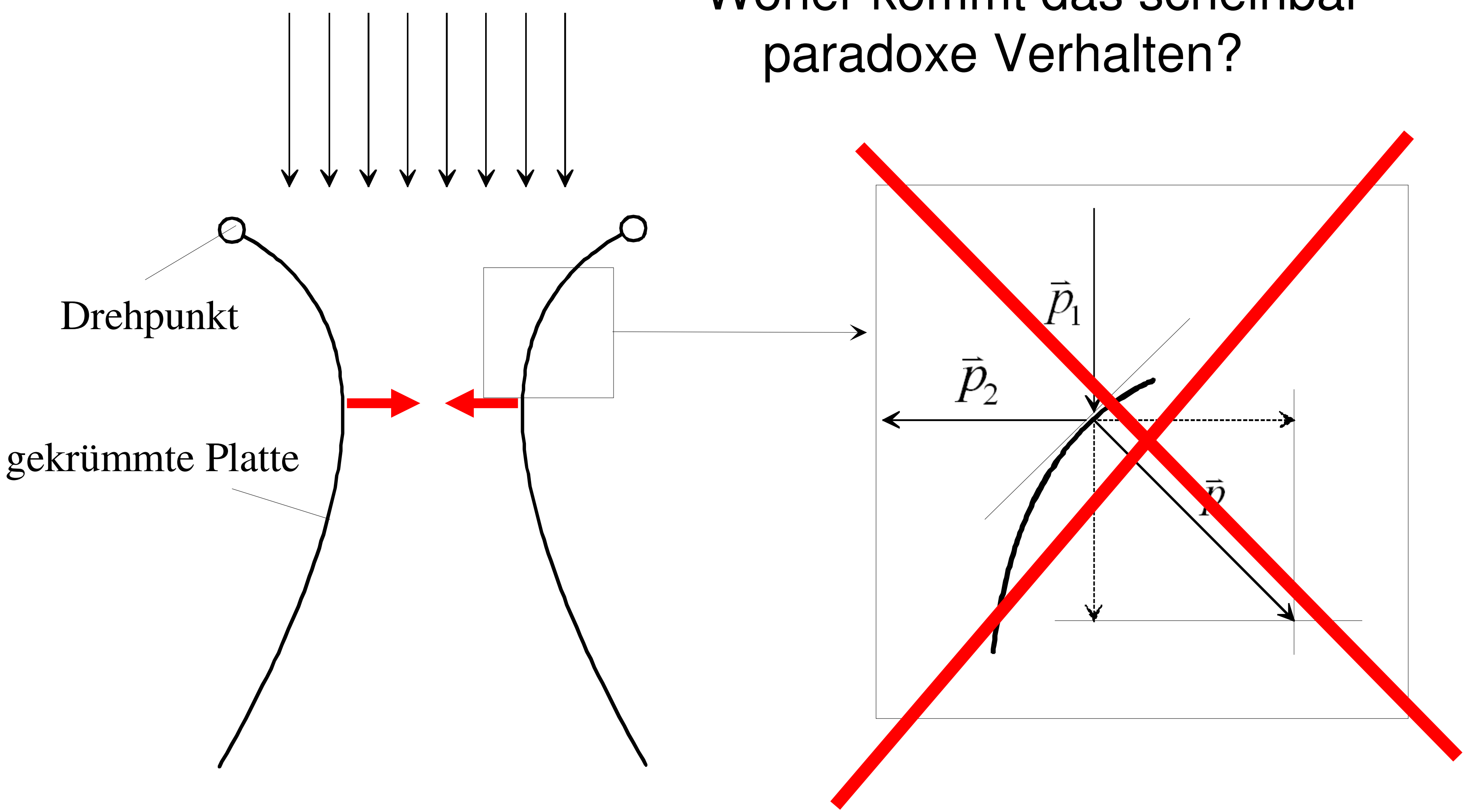
Prolog: Ein paar Experimente

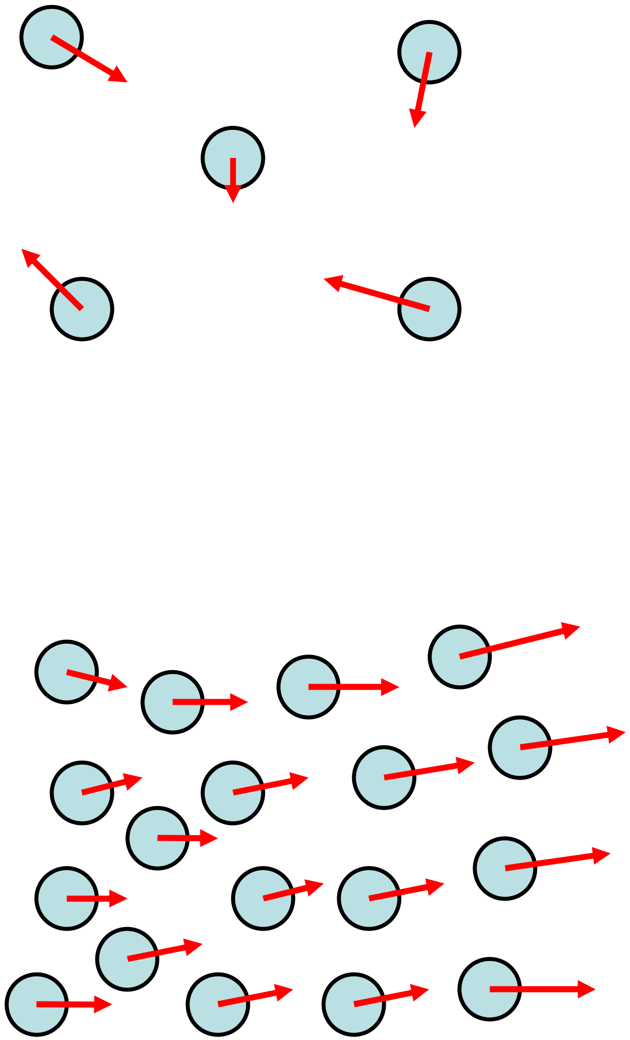




622 Theorie

Woher kommt das scheinbar paradoxe Verhalten?





622 Theorie

Woher kommt das scheinbar paradoxe Verhalten?

Richtiges Modell anwenden!

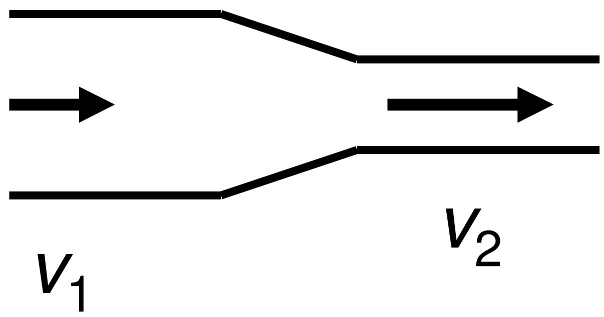
Verhalten von Gasen bei **sehr tiefen** Drücken durch Partikel-Kinetik beschreibbar

Gas bei grösseren Drücken: Wechselwirkungen zwischen Molekülen führe zu einem makroskopischen Verhalten als **Kontinuum**

→ **Strömung!**

622 Theorie

Strömung in einem Rohr:
→ Kontinuitätsgleichung

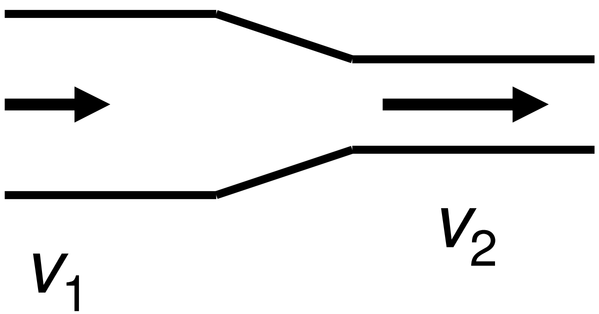


$$\frac{dV_1}{dt} = A_1 v_1 = \frac{dV_2}{dt} = A_2 v_2$$

622 Theorie

Strömung in einem Rohr:

→ Kontinuitätsgleichung



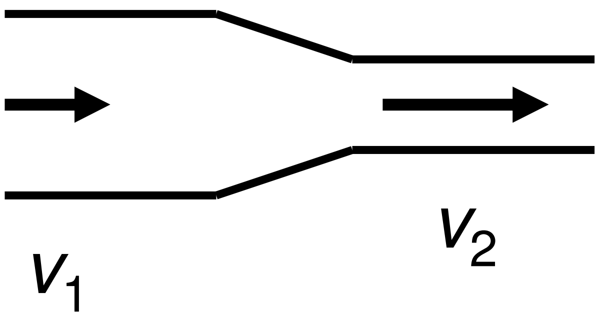
$$\frac{dV_1}{dt} = A_1 v_1 = \frac{dV_2}{dt} = A_2 v_2$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

622 Theorie

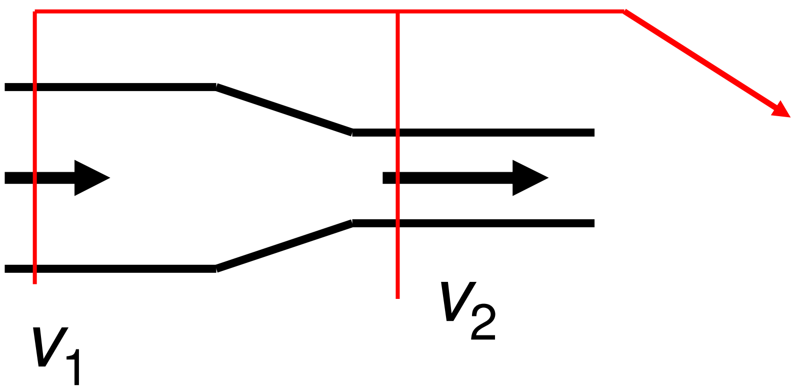
Energiebetrachtung entlang der Strömung

$$dW = F \cdot ds = pA \cdot v \cdot dt$$



622 Theorie

Energiebetrachtung entlang der Strömung



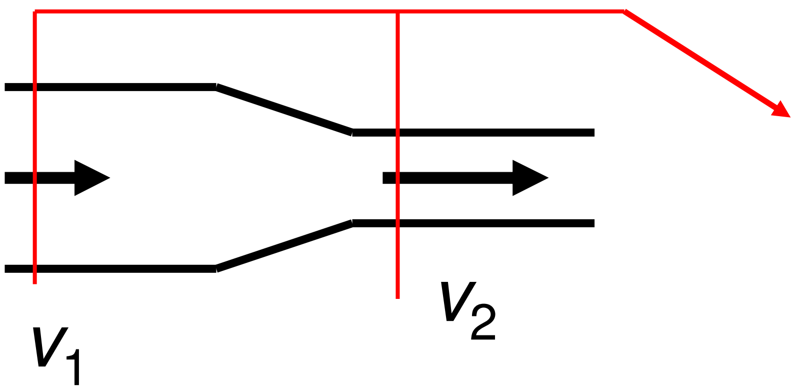
$$dW = F \cdot ds = pA \cdot v \cdot dt$$

$$\Delta(dW) =$$

$$= (p_1 A_1 v_1 - p_2 A_2 v_2) \cdot dt$$

622 Theorie

Energiebetrachtung entlang der Strömung



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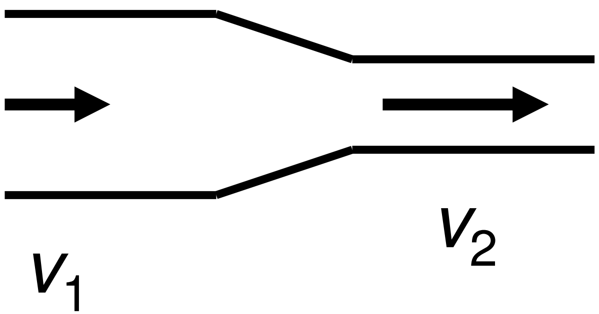
$$\Delta(dW) = (p_1 - p_2) A_1 v_1 \cdot dt$$

622 Theorie

kinetische Energie:

$$\Delta(dW) = (p_1 - p_2)A_1 v_1 \cdot dt$$

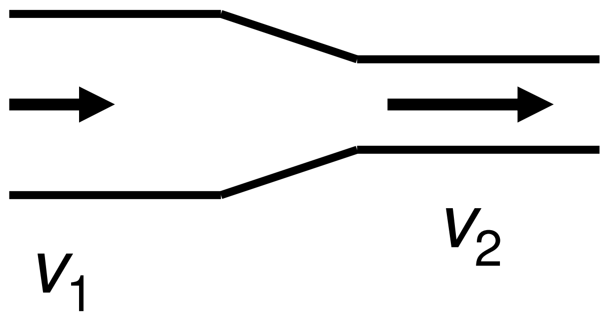
$$\Delta E_{kin} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$



622 Theorie

kinetische Energie:

$$\Delta(dW) = (p_1 - p_2)A_1 v_1 \cdot dt$$



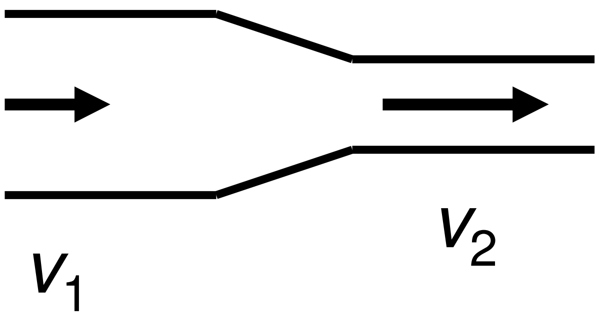
$$\Delta E_{kin} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= \frac{1}{2} \rho \cdot dV_2 \cdot v_2^2 - \frac{1}{2} \rho \cdot dV_1 \cdot v_1^2$$

622 Theorie

kinetische Energie:

$$\Delta(dW) = (p_1 - p_2)A_1 v_1 \cdot dt$$



$$\Delta E_{kin} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

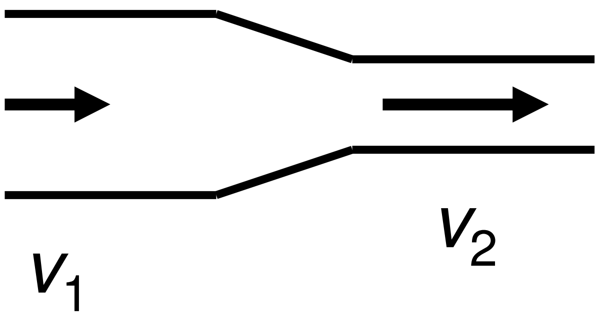
$$= \frac{1}{2}\rho \cdot dV_2 \cdot v_2^2 - \frac{1}{2}\rho \cdot dV_1 \cdot v_1^2$$

$$= \frac{1}{2}\rho \cdot A_2 \cdot v_2 dt \cdot v_2^2 - \frac{1}{2}\rho \cdot A_1 \cdot v_1 dt \cdot v_1^2$$

622 Theorie

kinetische Energie:

$$\Delta(dW) = (p_1 - p_2)A_1 v_1 \cdot dt$$



$$\Delta E_{kin} =$$

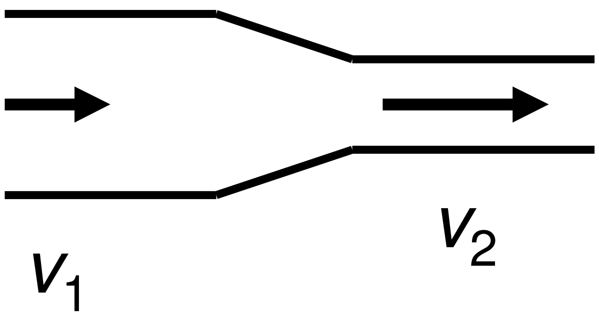
$$= \frac{1}{2}\rho \cdot A_2 \cdot v_2 dt \cdot v_2^2 - \frac{1}{2}\rho \cdot A_1 \cdot v_1 dt \cdot v_1^2$$

$$\Delta E_{kin} = \frac{1}{2}\rho A_1 v_1 dt(v_2^2 - v_1^2)$$

622 Theorie

Energieerhaltung

$$\boxed{\Delta(dW) = (p_1 - p_2) A_1 v_1 \cdot dt} = \boxed{\Delta E_{kin} = \frac{1}{2}\rho A_1 v_1 dt (v_2^2 - v_1^2)}$$

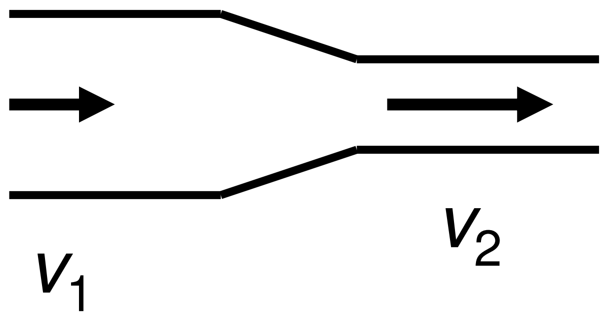


$$\rightarrow \frac{1}{2}\rho(v_2^2 - v_1^2) = p_1 - p_2$$

622 Theorie

Energieerhaltung

$$\boxed{\Delta(dW) = (p_1 - p_2)A_1 v_1 \cdot dt} = \boxed{\Delta E_{kin} = \frac{1}{2}\rho A_1 v_1 dt (v_2^2 - v_1^2)}$$



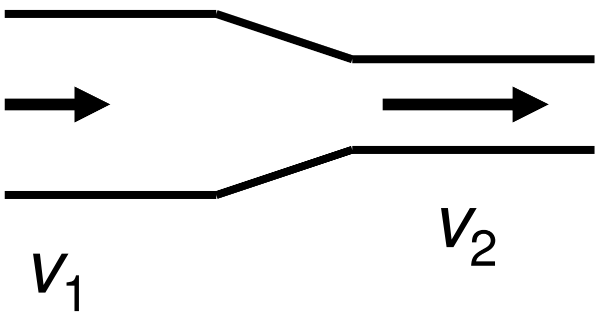
$$\frac{1}{2}\rho(v_2^2 - v_1^2) = p_1 - p_2$$

$$\longrightarrow \boxed{p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2}$$

622 Theorie

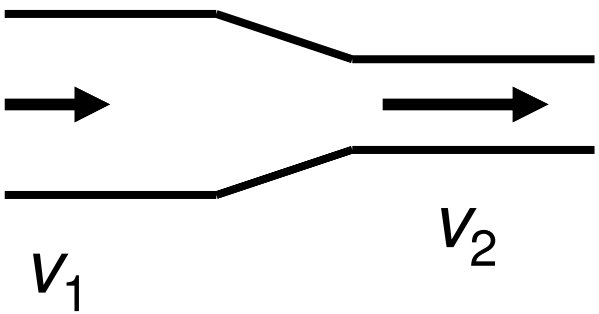
Recap Energiebetrachtung mit potentieller Energie

$$\frac{d}{dV}\left[E_{kin}+E_{pot}\right]=\frac{d}{dV}\left[\frac{1}{2}mv^2\right]+\frac{d}{dV}\left[mgh\right]$$



622 Theorie

Recap Energiebetrachtung mit potentieller Energie

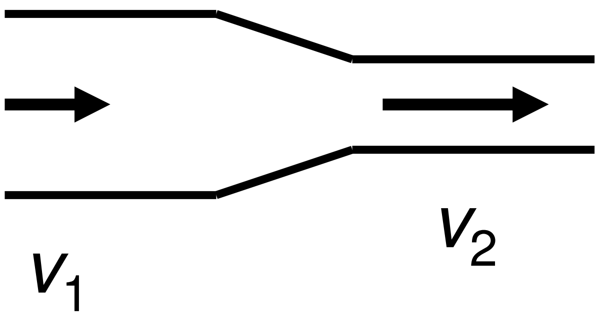


$$\frac{d}{dV}\left[E_{kin}+E_{pot}\right]=\frac{d}{dV}\left[\frac{1}{2}mv^2\right]+\frac{d}{dV}\left[mgh\right]$$

$$=\frac{1}{2}v^2\frac{dm}{dV}+gh\frac{dm}{dV}$$

622 Theorie

Recap Energiebetrachtung mit potentieller Energie



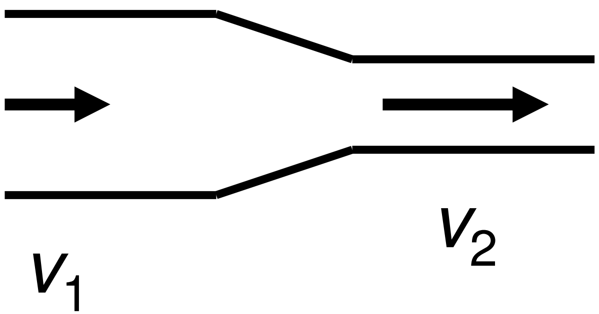
$$\frac{d}{dV}\left[E_{kin}+E_{pot}\right]=\frac{d}{dV}\left[\frac{1}{2}mv^2\right]+\frac{d}{dV}\left[mgh\right]$$

$$=\frac{1}{2}v^2\frac{dm}{dV}+gh\frac{dm}{dV}=\frac{1}{2}\rho v^2+\rho gh$$

622 Theorie

Recap Energiebetrachtung mit potentieller Energie

$$\frac{d}{dV}\left[E_{kin}+E_{pot}\right]=\frac{d}{dV}\left[\frac{1}{2}mv^2\right]+\frac{d}{dV}\left[mgh\right]$$



$$=\frac{1}{2}v^2\frac{dm}{dV}+gh\frac{dm}{dV}=\frac{1}{2}\rho v^2+\rho gh$$

$$\left[\frac{1}{2}\rho v_1^2+\rho g h_1\right]\cdot dt=\left[\frac{1}{2}\rho v_2^2+\rho g h_2\right]\cdot dt=const.$$

622 Theorie

Recap Energiebetrachtung mit potentieller Energie

$$\left[\frac{1}{2}\rho v_1^2 + \rho g h_1\right]\cdot dt = \left[\frac{1}{2}\rho v_2^2 + \rho g h_2\right]\cdot dt = const.$$

$$\Rightarrow \quad \frac{1}{2}\rho v_1^2 + \rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

622 Theorie

Recap Energiebetrachtung mit potentieller Energie

$$\left[\frac{1}{2}\rho v_1^2 + \rho g h_1\right]\cdot dt = \left[\frac{1}{2}\rho v_2^2 + \rho g h_2\right]\cdot dt = const.$$

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

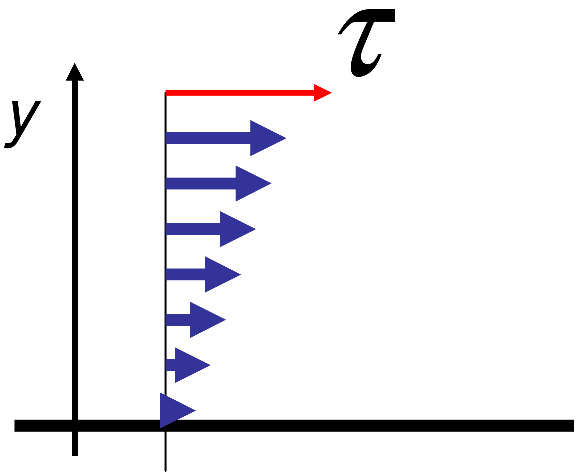
$$\Rightarrow \quad \frac{1}{2} v_1^2 + g h_1 = \frac{1}{2} v_2^2 + g h_2$$

623 stationäre Strömung mit Reibung

623 Ziele

- Strömungswiderstand in Rohrleitungen berechnen können
- Reynoldszahl für Rohrströmung berechnen und daraus Schlüsse bezüglich Strömungsart ziehen können
- stationäre und instationäre (turbulente) Strömung charakterisieren können

623 Theorie



Schub- / Scherspannung und Viskosität

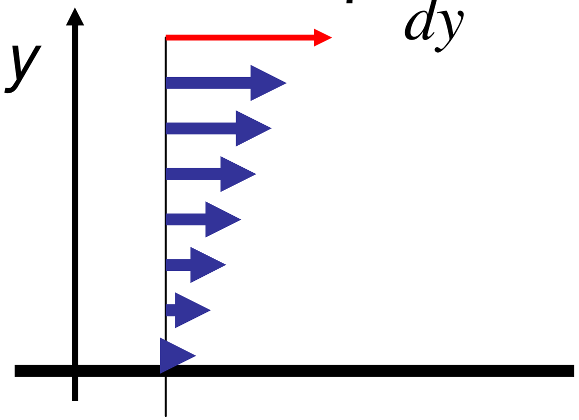
$$\tau = \eta \cdot \frac{dv}{dy}$$

Tab.1. dynamische Viskositäten für einige Fluide

Medium	$\eta / Pa \cdot s$
Wasser	$\approx 10^{-3}$
Luft	$\approx 2 \cdot 10^{-5}$
Schmieröl	$\approx 0.1 - 1$
Glycerin	≈ 1.5

623 Theorie

$$\tau = \eta \cdot \frac{dv}{dy}$$

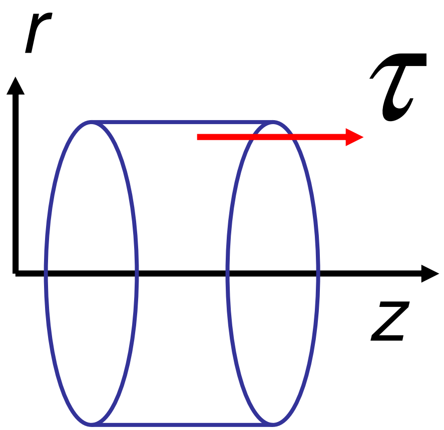


kinematische Viskosität

$$\nu = \frac{\eta}{\rho}$$

623 Theorie

Strömungsprofil in einem Rohr

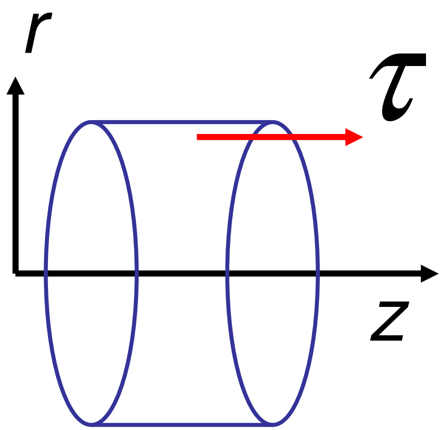


$$A_z = 2\pi r \cdot \Delta z$$

$$F_R = A_z \cdot \tau = A_z \eta \cdot \frac{dv}{dr}$$

623 Theorie

Antreibende Kraft *F*

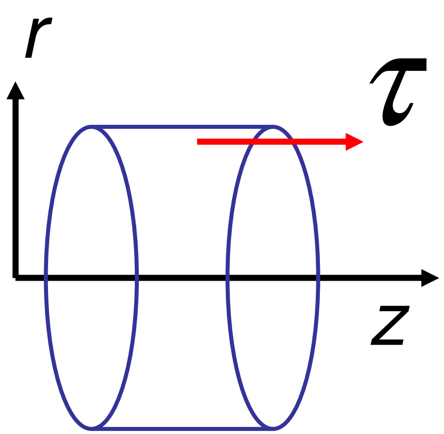


$$A_z = 2\pi r \cdot \Delta z$$

$$F = \pi r^2 \cdot \Delta p$$

$$F_R = A_z \cdot \tau = A_z \eta \cdot \frac{dv}{dr}$$

623 Theorie



$A_z = 2\pi r \cdot \Delta z$

Antreibende Kraft *F*, im stationären Zustand gleich Reibungskraft

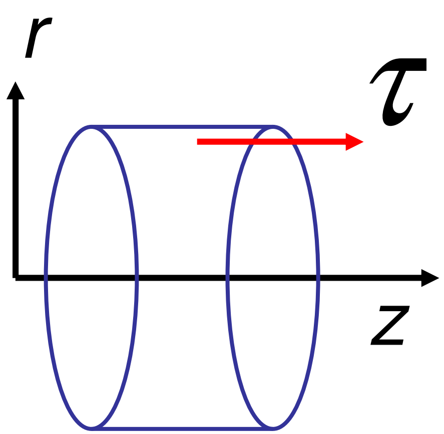
$$\boxed{F = \pi r^2 \cdot \Delta p} =$$

$$\boxed{F_R = A_z \cdot \tau = A_z \eta \cdot \frac{dv}{dr}}$$

$$\rightarrow \quad \frac{dv}{dr} = \frac{\pi r^2}{\eta A_z} \cdot \Delta p = \frac{r}{2\eta} \cdot \frac{\Delta p}{\Delta z}$$

623 Theorie

radiale Integration



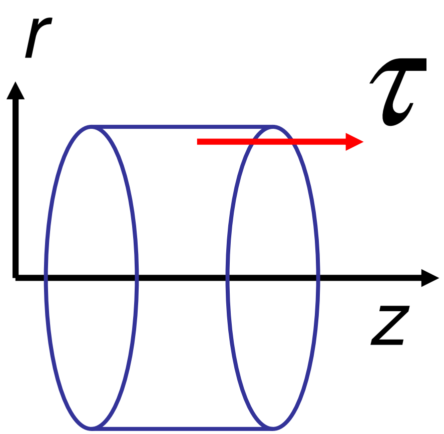
$$A_z = 2\pi r \cdot \Delta z$$

$$\frac{dv}{dr} = \frac{\pi r^2}{\eta A_z} \cdot \Delta p = \frac{r}{2\eta} \cdot \frac{\Delta p}{\Delta z}$$

$$\Rightarrow \quad v(r) = \frac{\Delta p}{2\eta \cdot \Delta z} \int_r^R r \cdot dr$$

623 Theorie

radiale Integration



$A_z = 2\pi r \cdot \Delta z$

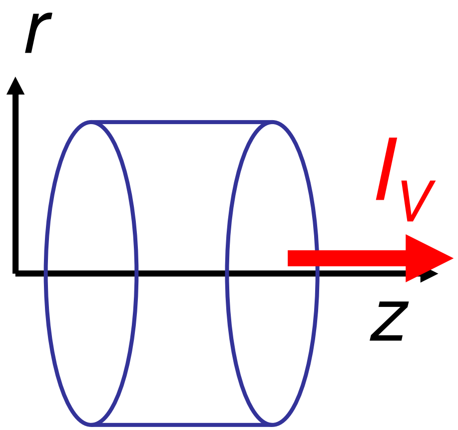
$$\frac{dv}{dr} = \frac{\pi r^2}{\eta A_z} \cdot \Delta p = \frac{r}{2\eta} \cdot \frac{\Delta p}{\Delta z}$$

$$\Rightarrow \quad v(r) = \frac{\Delta p}{2\eta \cdot \Delta z} \int_r^R r \cdot dr$$

$$= \frac{\Delta p}{4\eta \cdot \Delta z} \cdot \left(R^2 - r^2\right)$$

623 Theorie

Volumenstrom

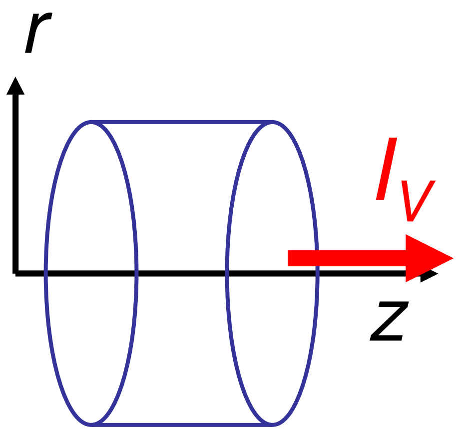


$$v(r) = \frac{\Delta p}{4\eta \cdot \Delta z} \cdot \left(R^2 - r^2\right)$$

$$d(\Delta V) = 2\pi r \cdot dr \cdot v(r) \cdot \Delta t$$

623 Theorie

Volumenstrom



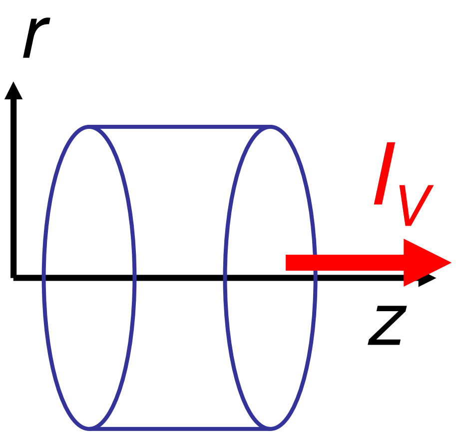
$$v(r) = \frac{\Delta p}{4\eta \cdot \Delta z} \cdot \left(R^2 - r^2\right)$$

$$d(\Delta V) = 2\pi r \cdot dr \cdot v(r) \cdot \Delta t$$

$$\Delta V = \frac{\pi R^4}{8\eta} \cdot \frac{\Delta p}{\Delta z} \cdot \Delta t$$

623 Theorie

Volumenstrom



$$v(r) = \frac{\Delta p}{4\eta \cdot \Delta z} \cdot \left(R^2 - r^2\right)$$

$$d(\Delta V) = 2\pi r \cdot dr \cdot v(r) \cdot \Delta t$$

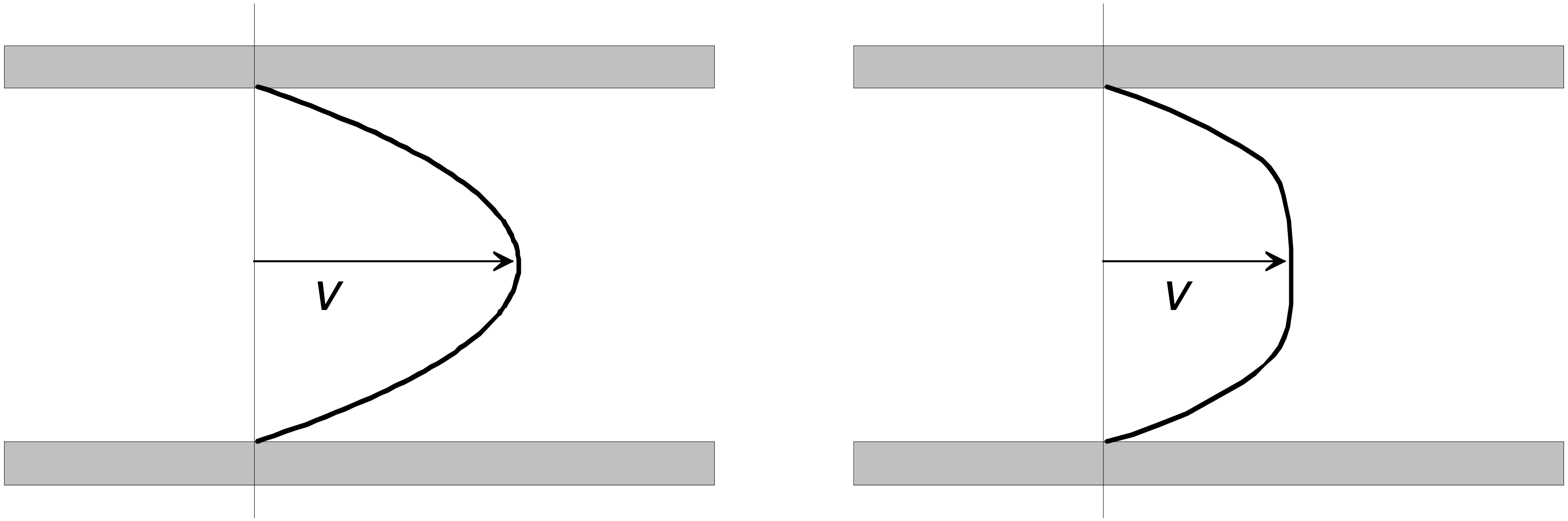
$$\Delta V = \frac{\pi R^4}{8\eta} \cdot \frac{\Delta p}{\Delta z} \cdot \Delta t$$

$$\frac{\Delta V}{\Delta t} = \frac{\pi R^4}{8\eta} \cdot \frac{\Delta p}{\Delta z}$$

$$= I_V = \frac{\Delta p}{R_V}$$

623 Theorie

Strömungsprofil für stationäre und turbulente Strömung

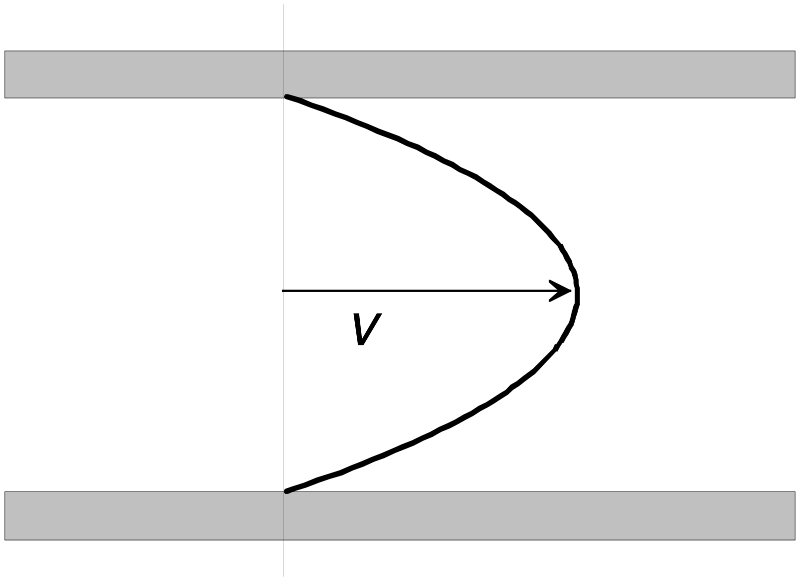


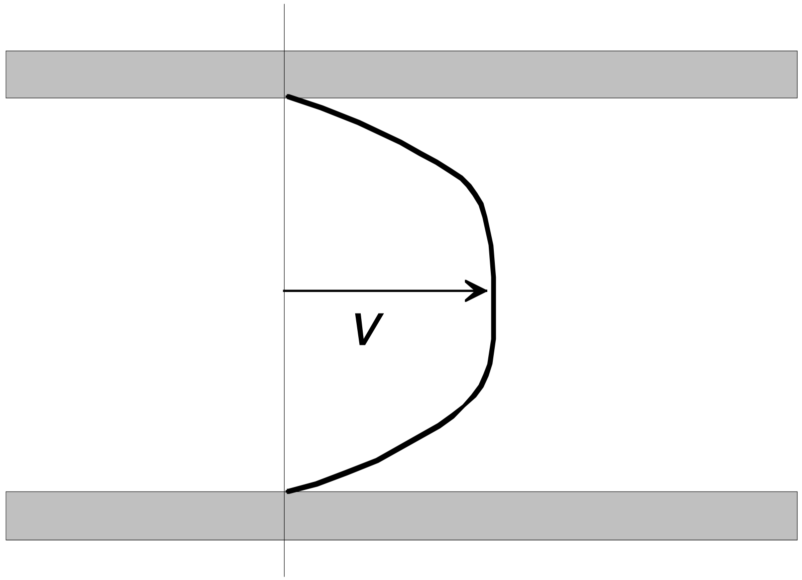
623 Theorie

$$\mathrm{Re} = \frac{d \cdot \rho}{\eta} \cdot \overline{v}$$

Kriterium für Strömungsverhalten: Reynolds-Zahl

d = Rohrdurchmesser





623 Theorie

$$\frac{\Delta p}{\Delta z} = \frac{\lambda}{d} \cdot \frac{\rho \bar{v}^2}{2}$$

Druckgradient und Volumenstrom im turbulenten Fall

$$\bar{v} = \frac{I_V}{\pi \cdot (d/2)^2}$$

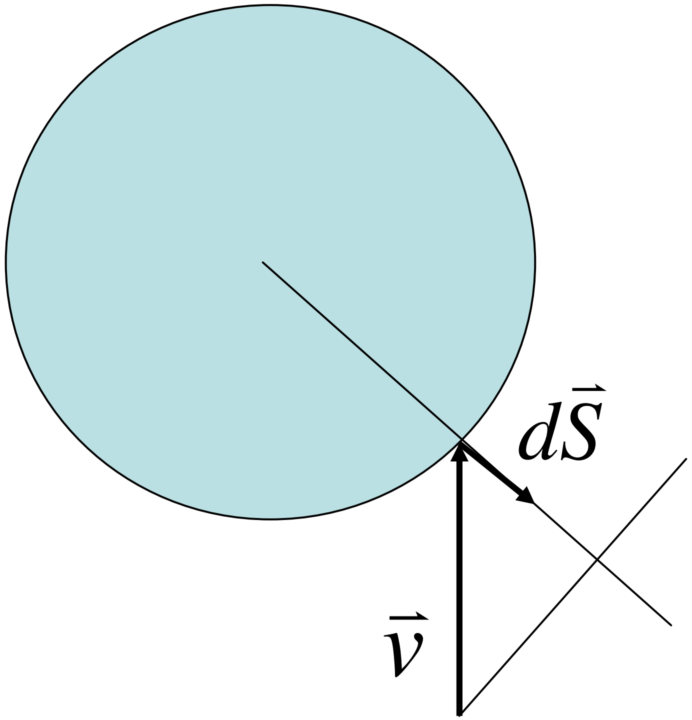
$$\Delta p = R \cdot I_V = \lambda \cdot \frac{8\rho \cdot \Delta z}{\pi^2 d^5} \cdot I_V^2$$

624 Laminare und turbulente Umströmung von Körpern

624 Ziele

- Widerstandsgesetze für laminare und turbulente Umströmung kennen und anwenden können
- Unterschiede bezüglich Geschwindigkeits-abhängigkeit auswendig benennen können

624 Theorie

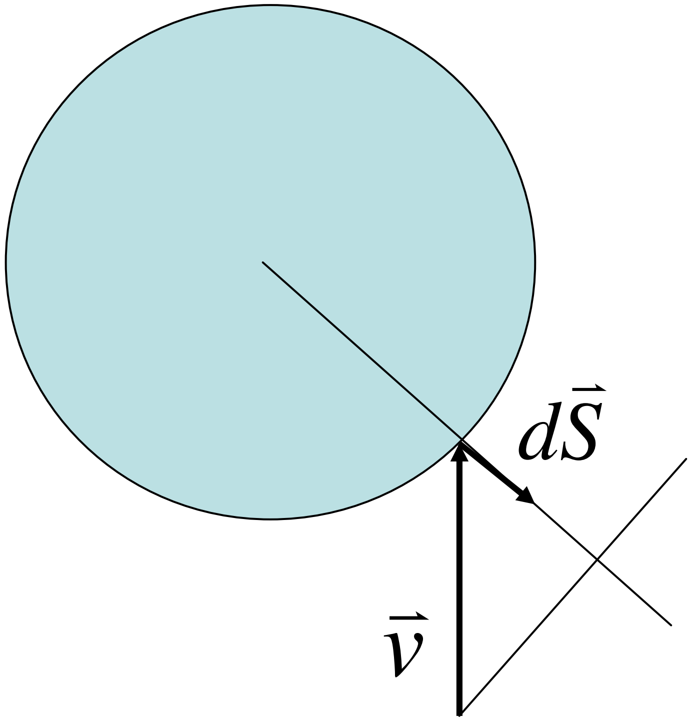


Reibung - Stokes

$$dF_R = \tau \cdot dS$$

$$d\vec{F}_R \bullet \frac{\vec{v}}{v} = \tau d\vec{S} \bullet \frac{\vec{v}}{v}$$

624 Theorie

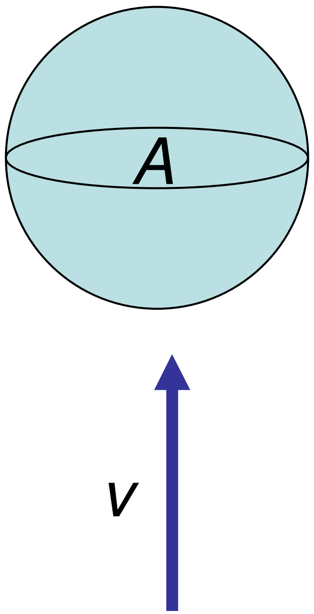


Reibung - Stokes

$$dF_R = \tau \cdot dS$$

$$d\vec{F}_R \bullet \frac{\vec{v}}{v} = \tau d\vec{S} \bullet \frac{\vec{v}}{v}$$

$$\boxed{F_R = 6\pi r \eta \cdot v}$$



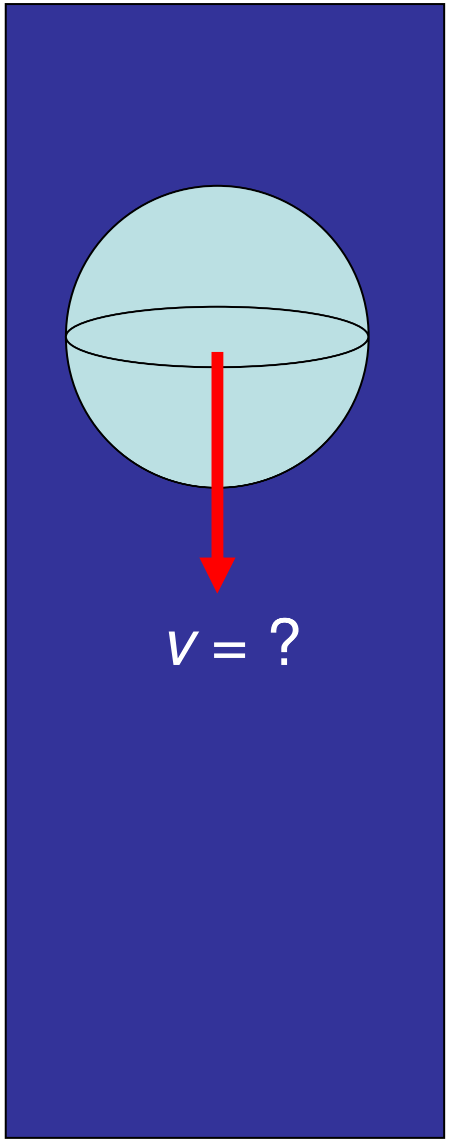
624 Theorie

Turbulenter Strömungswiderstand:

Staudruck * Korrekturfaktor

$$F_W = \frac{1}{2}\rho v^2 \cdot A$$

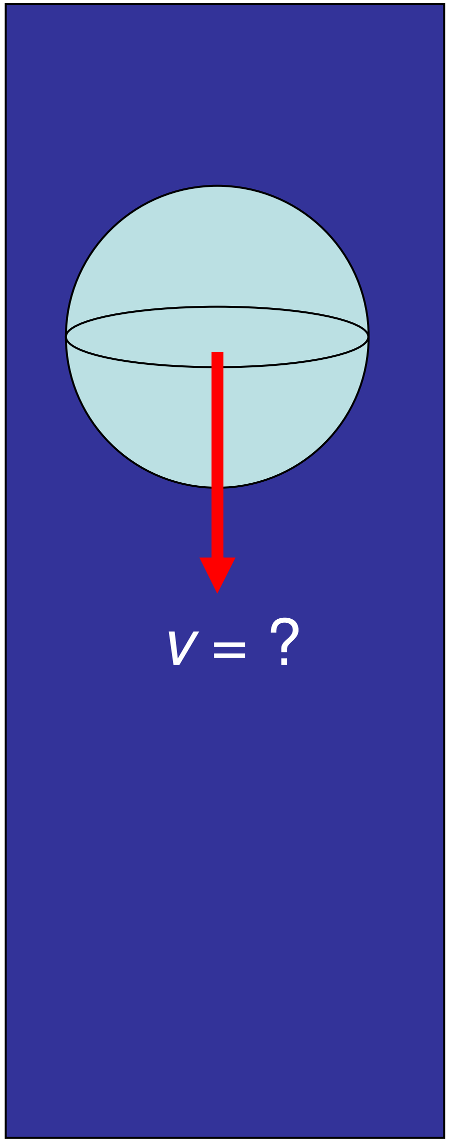
$$\boxed{F_W = c_w \cdot \frac{\rho A}{2} \cdot v^2}$$



624 Experiment und Aufgaben

Sinkgeschwindigkeit von Kugeln in viskoser Flüssigkeit

$$ma = mg - \rho_{fluid} \cdot Vg - 6\pi r \eta v$$

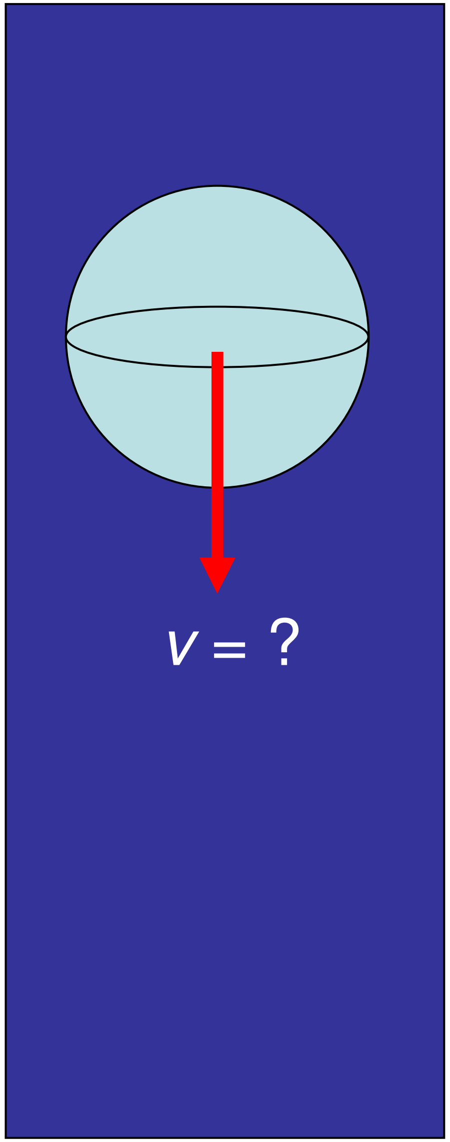


624 Experiment und Aufgaben

Sinkgeschwindigkeit von Kugeln in viskoser Flüssigkeit

$$ma = mg - \rho_{fluid} \cdot Vg - 6\pi r\eta v$$

$$\frac{dv}{dt} = \left(1 - \frac{\rho_{fluid} \cdot V}{m}\right) \cdot g - \frac{6\pi r\eta}{m} \cdot v$$



624 Experiment und Aufgaben

Sinkgeschwindigkeit von Kugeln in viskoser Flüssigkeit

$$ma = mg - \rho_{fluid} \cdot Vg - 6\pi r \eta v$$

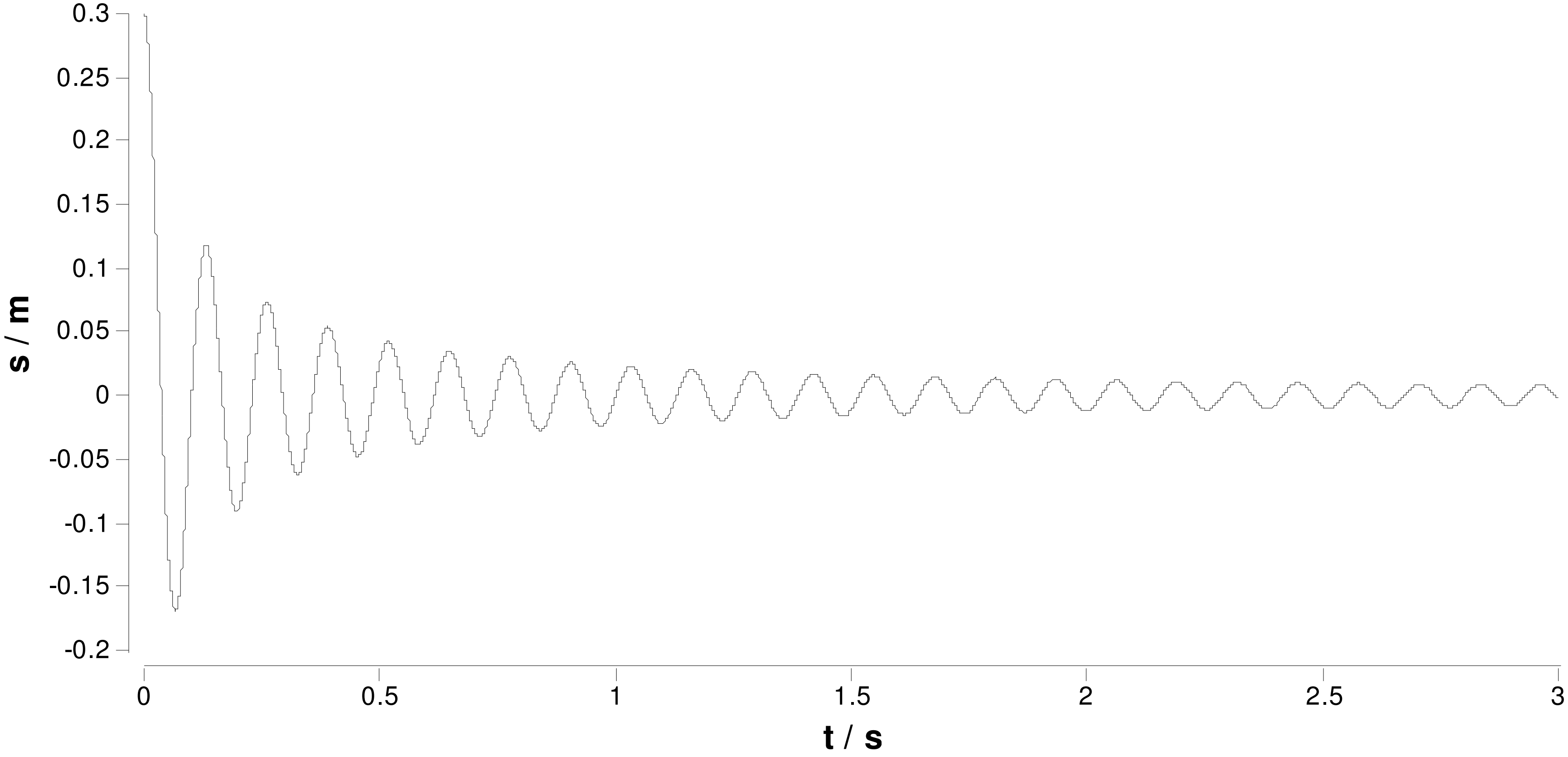
$$\frac{dv}{dt} = \left(1 - \frac{\rho_{fluid} \cdot V}{m}\right) \cdot g - \frac{6\pi r \eta}{m} \cdot v$$

$$v_{eq} = \frac{\left(m - \rho_{fluid} \cdot V\right) \cdot g}{6\pi r \eta}$$

624 Aufgaben

Unterschiedliche Dämpfung

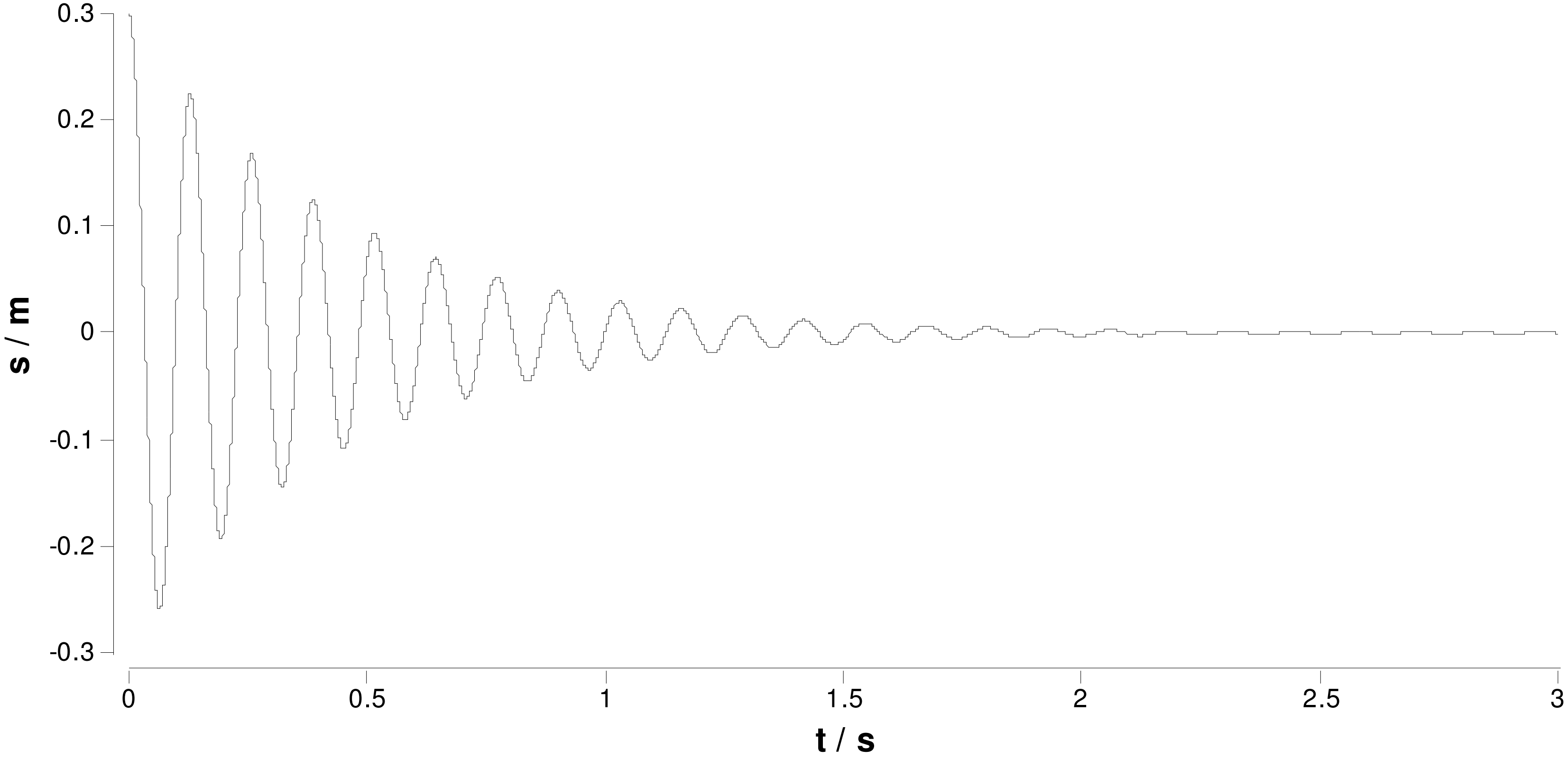
Turbulent: 1/*t* - Hüllkurve



624 Aufgaben

Unterschiedliche Dämpfung

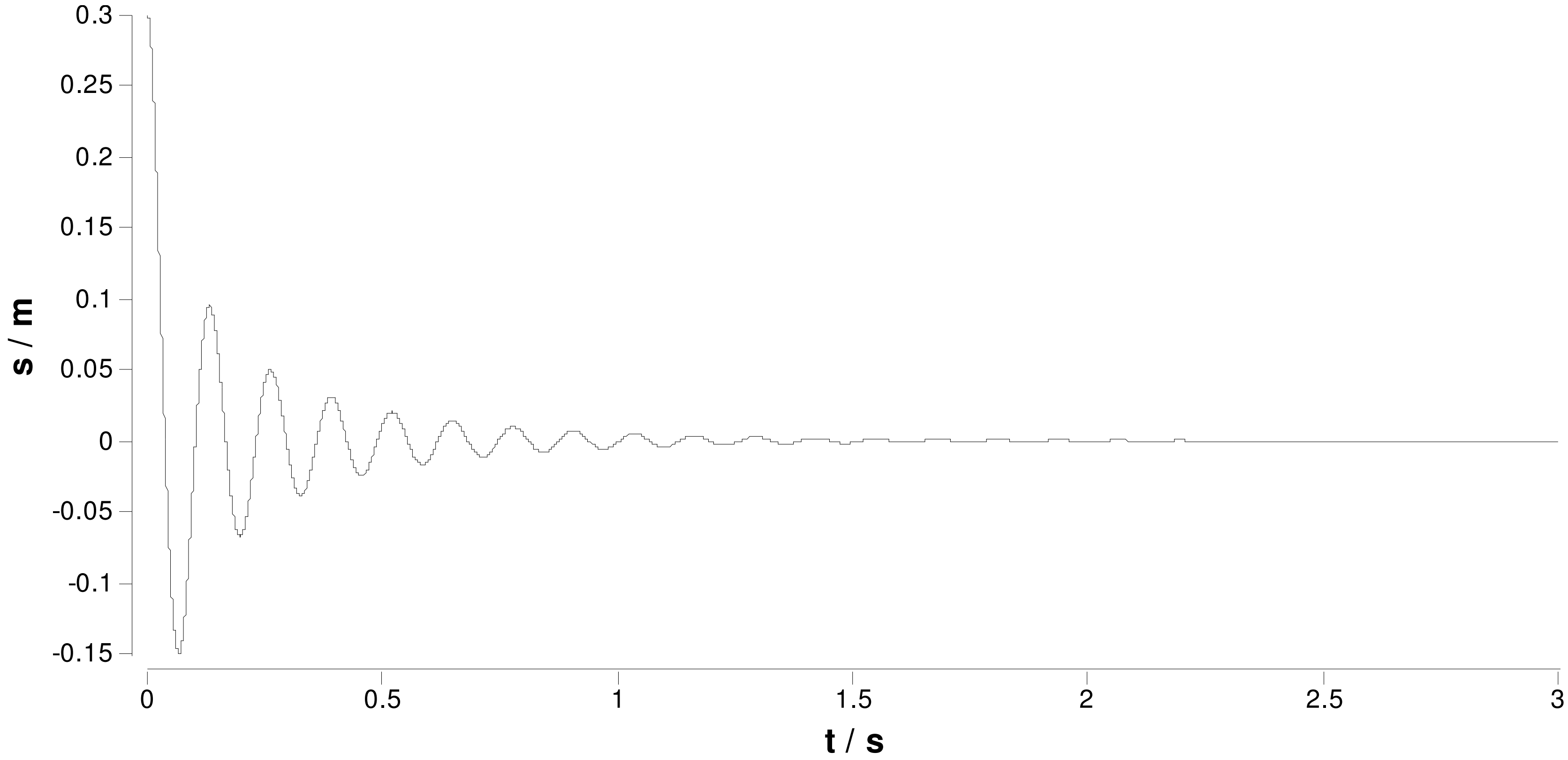
laminar: exponentielle Hüllkurve



624 Aufgaben

Unterschiedliche Dämpfung

real: Kombination



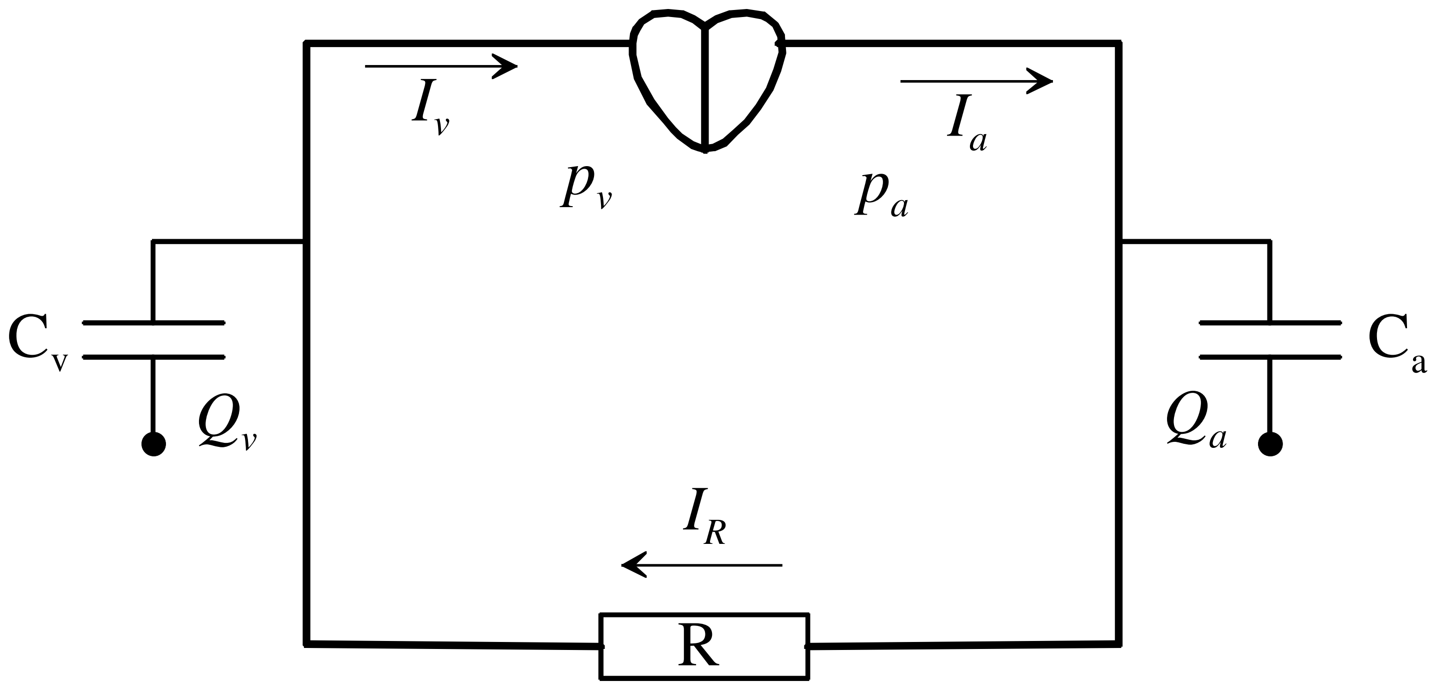
625 Simulation von Systemen mit Speichern und Flüssen

625 Ziele

- (hydraulische) Systeme mit kapazitiven und resistiven Elementen modellieren und simulieren können

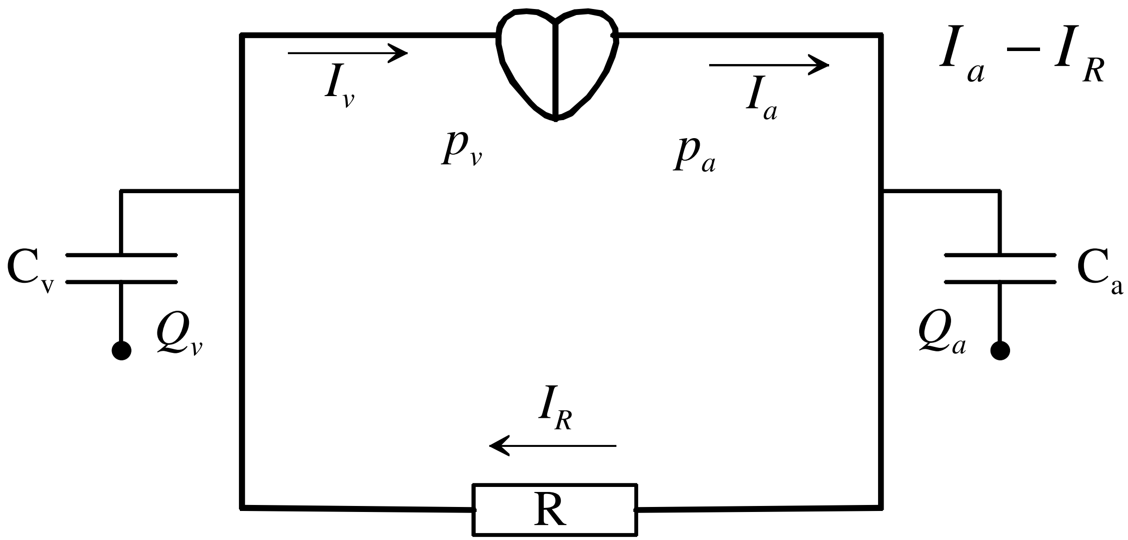
625 Theorie

Bsp. Blutkreislauf: Analogie zu elektrischen Schaltungen!



625 Theorie

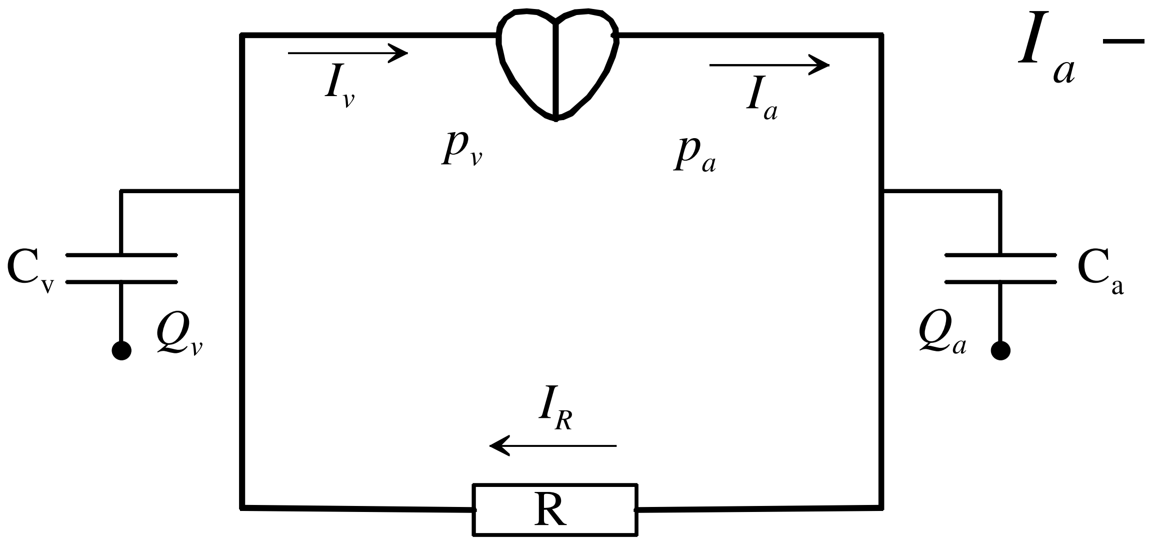
Bsp. Blutkreislauf: Analogie zu elektrischen Schaltungen!



$$I_a - I_R = \frac{dQ_a}{dt} = \frac{dQ_a}{dp_a} \cdot \frac{dp_a}{dt}$$

625 Theorie

Bsp. Blutkreislauf: Analogie zu elektrischen Schaltungen!



$$I_a - I_R = C_a \cdot \frac{dp_a}{dt}$$

$$C_a = \frac{dQ_a}{dp_a}$$

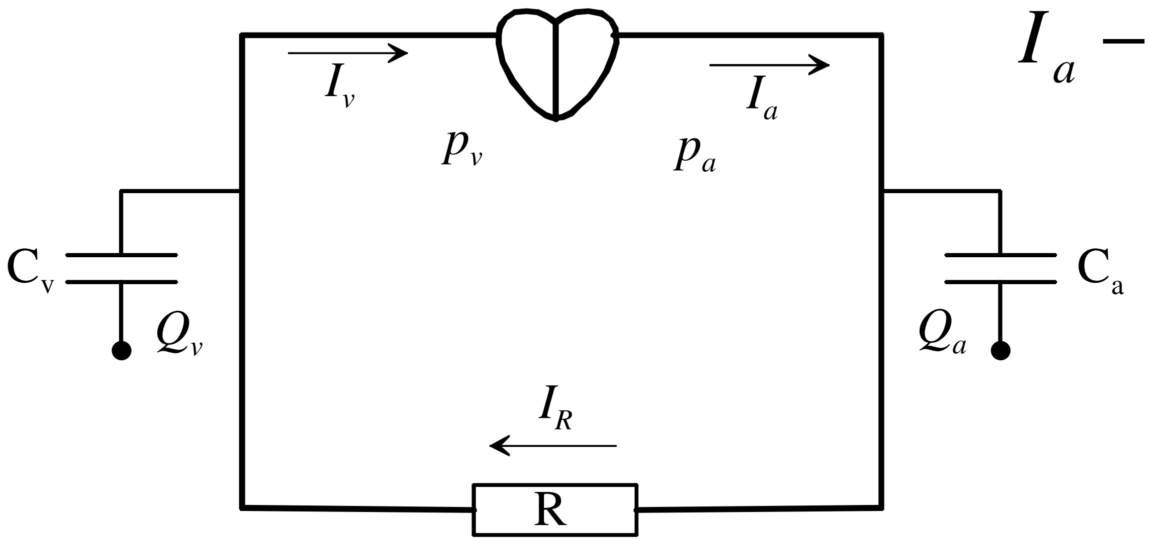
625 Theorie

Bsp. Blutkreislauf: Analogie zu elektrischen Schaltungen!

$$\boxed{R \cdot I_R = p_a - p_v}$$

$$I_R - I_v = C_v \cdot \frac{dp_v}{dt}$$

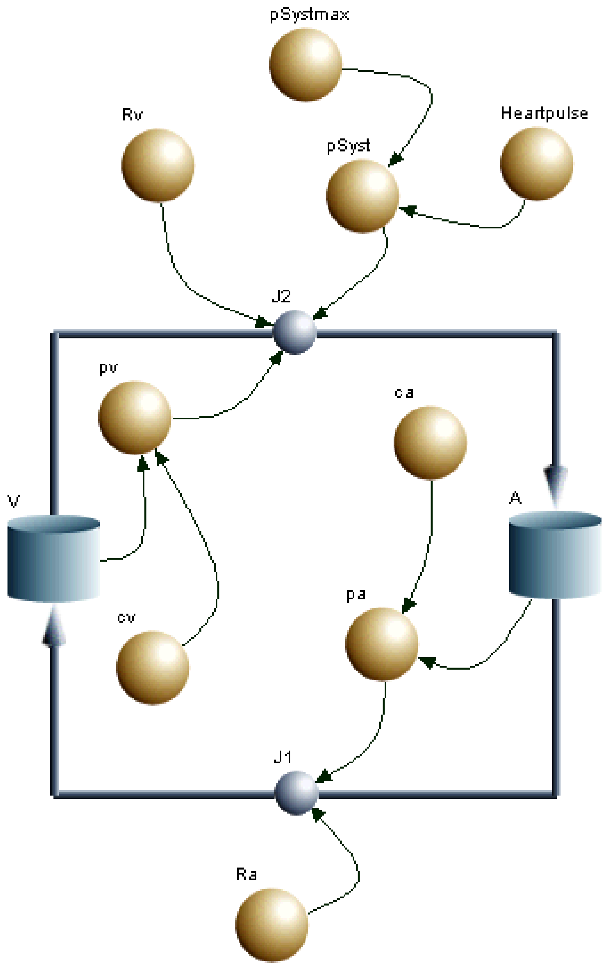
$$I_a - I_R = C_a \cdot \frac{dp_a}{dt}$$

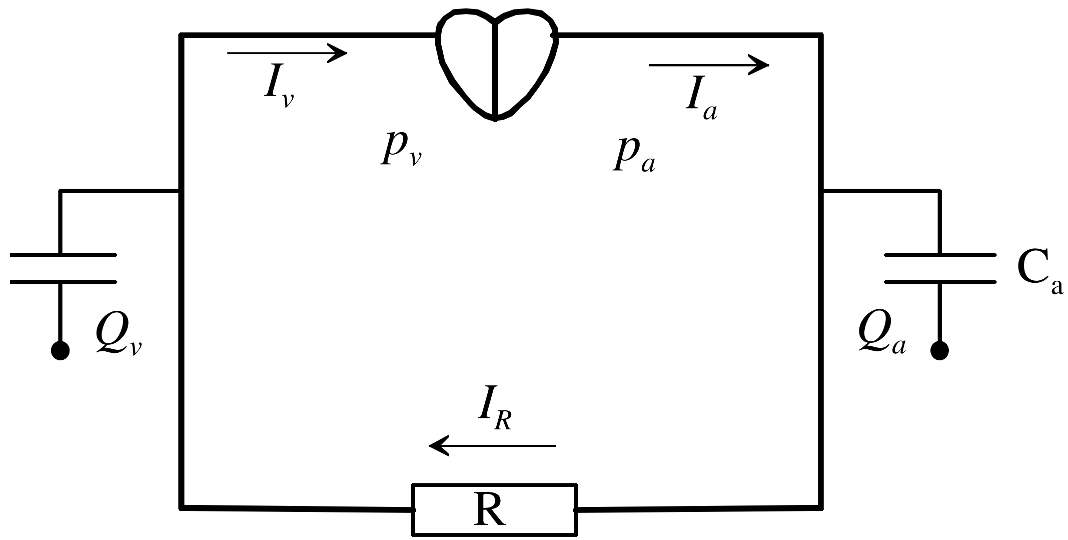


$$C_a = \frac{dQ_a}{dp_a}$$

625 Theorie

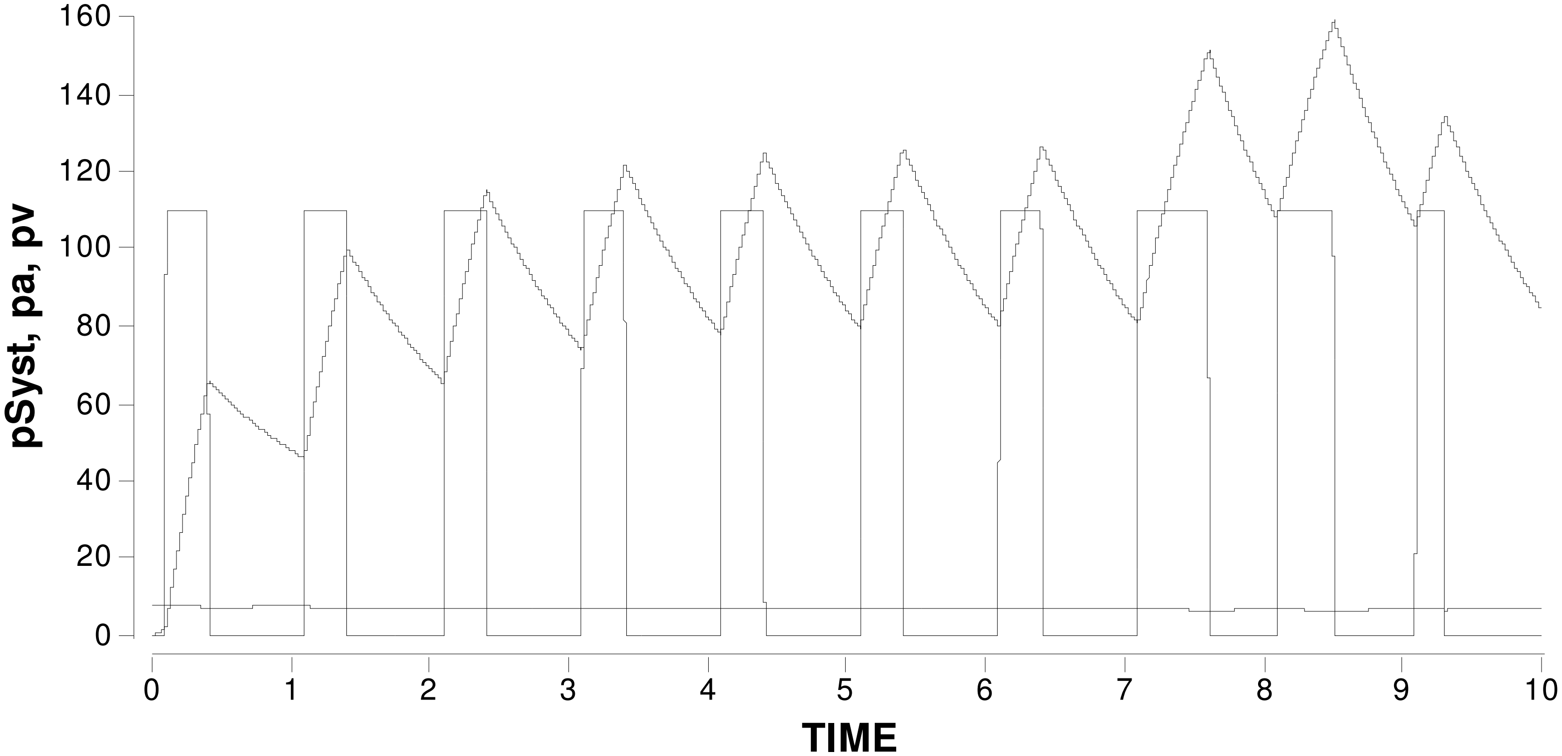
Bsp. Blutkreislauf: Analogie zu elektrischen Schaltungen!



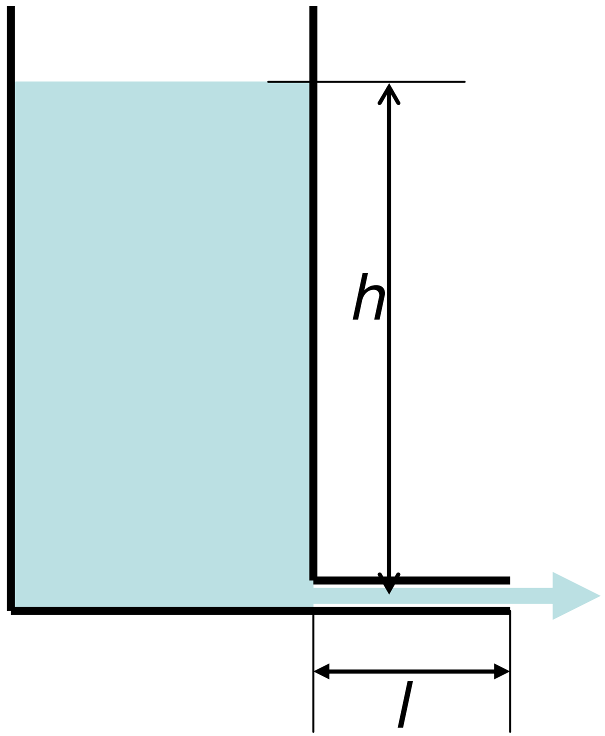


625 Theorie

Bsp. Blutkreislauf: Druckverlauf während Systole und Diastole



625 Theorie



Ausfliessender Speicher:

Volumenstrom und Widerstand

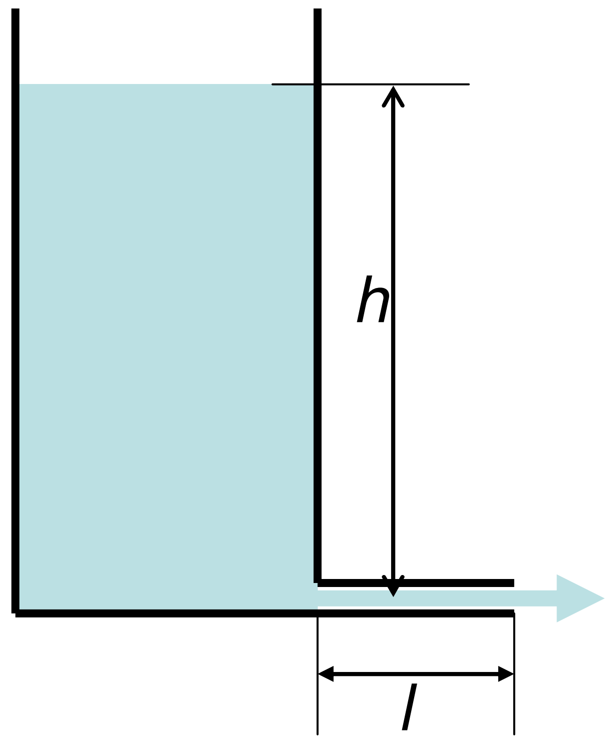
$$I_V = \frac{\Delta p}{R_V} = \frac{\pi r^4}{8\eta l} \cdot \Delta p$$

$$R_V = \frac{8\eta l}{\pi r^4}$$

$$\frac{dV}{dt} = -I_V = -\frac{\pi r^4}{8\eta l} \cdot \Delta p$$

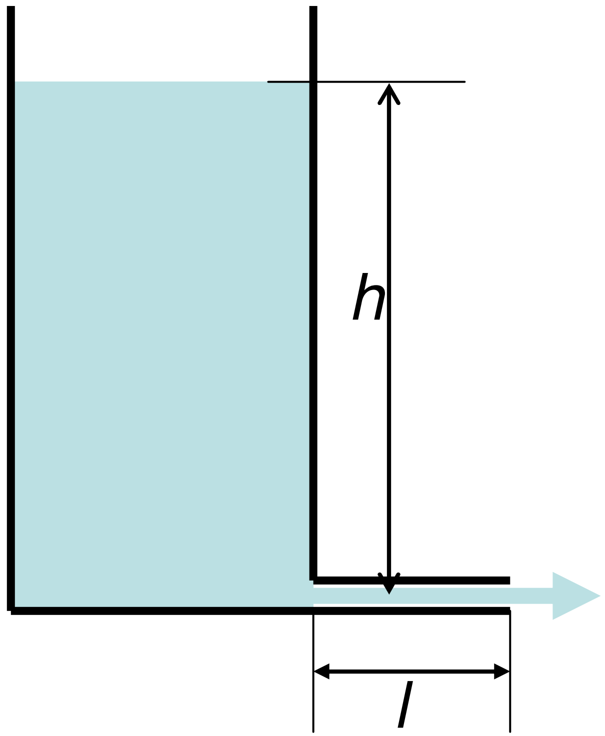
625 Theorie

Ausfliessender Speicher:
Volumenstrom und Widerstand



$$\frac{dV}{dt} = -I_V = -\frac{\pi r^4}{8\eta l} \cdot \Delta p$$

$$\Rightarrow \quad \frac{dh}{dt} = -\frac{\rho g \pi r^4}{8\eta l A} \cdot h$$



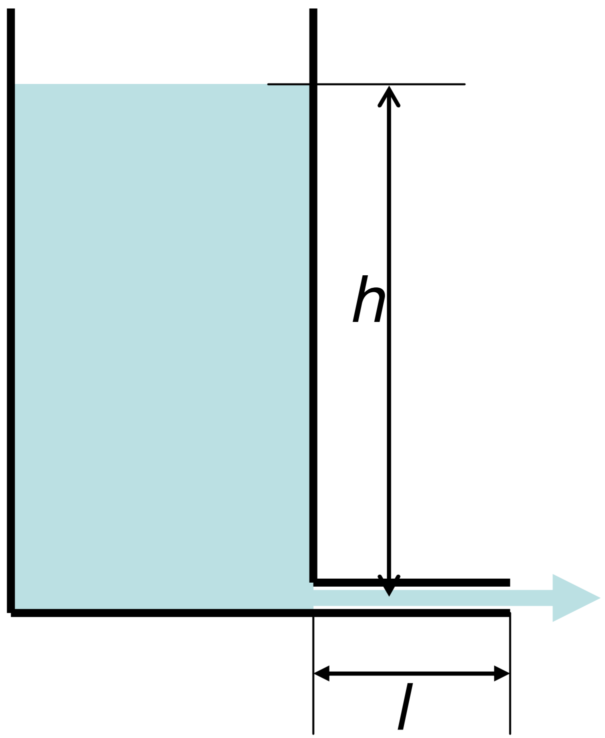
625 Theorie

Lösung $h(t)$=?

$$\frac{dh}{dt} = -\frac{\rho g \pi r^4}{8\eta l A} \cdot h$$

625 Theorie

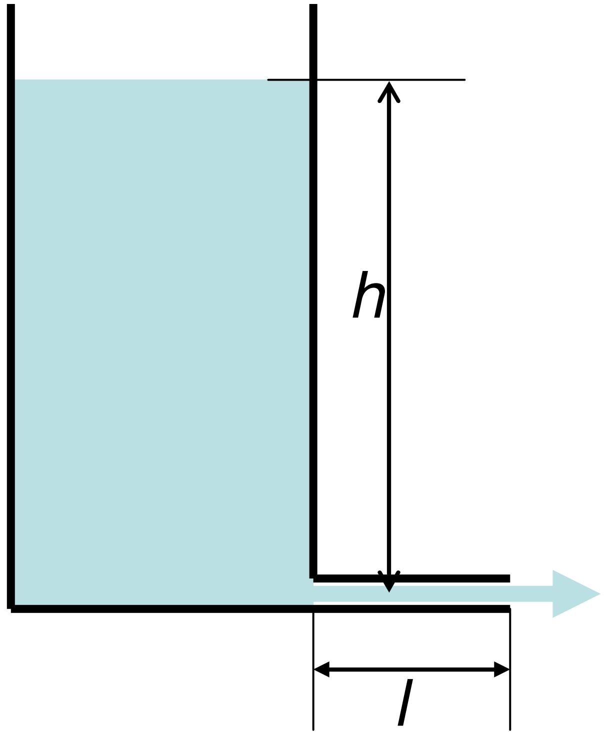
Lösung $h(t)$=?



$$\frac{dh}{dt} = -\frac{\rho g \pi r^4}{8\eta l A} \cdot h$$

$$\boxed{\begin{aligned} \dot{h} &= -h/\tau \\ h(t) &= h_0 \cdot e^{-t/\tau} \end{aligned}}$$

625 Theorie



Kapazität

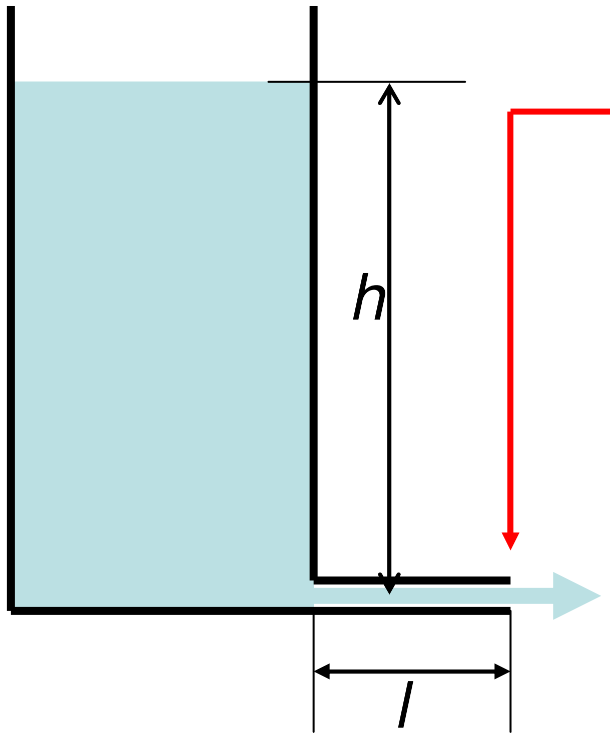
$$C = \frac{dV}{dp} = \frac{A \cdot dh}{\rho g \cdot dh} = \frac{A}{\rho g}$$

$$\frac{1}{\tau} = \frac{\rho g \pi r^4}{8 \eta l A}$$

$$\boxed{\tau = RC}$$

625 Theorie

Induktivität

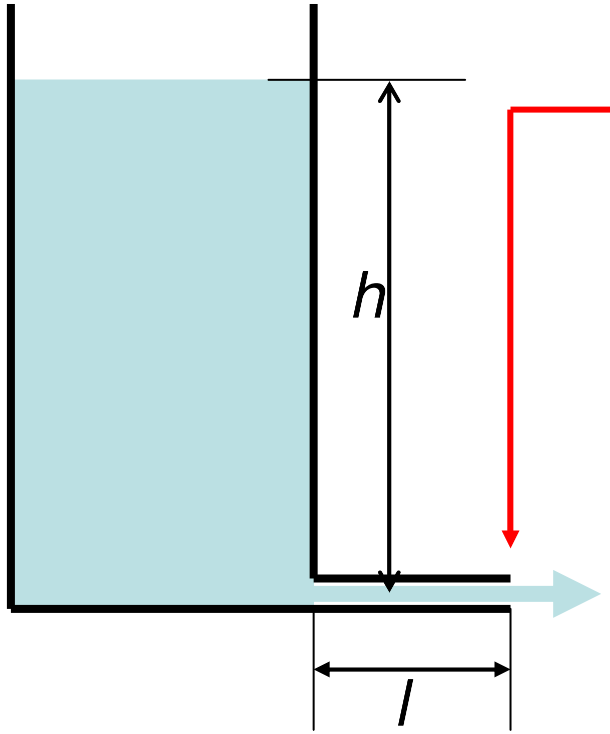


$$m \cdot a = V \cdot \rho \cdot \dot{v}$$

$$= A_p l \rho \cdot \dot{v} = -A_p \cdot \Delta p$$

625 Theorie

Induktivität



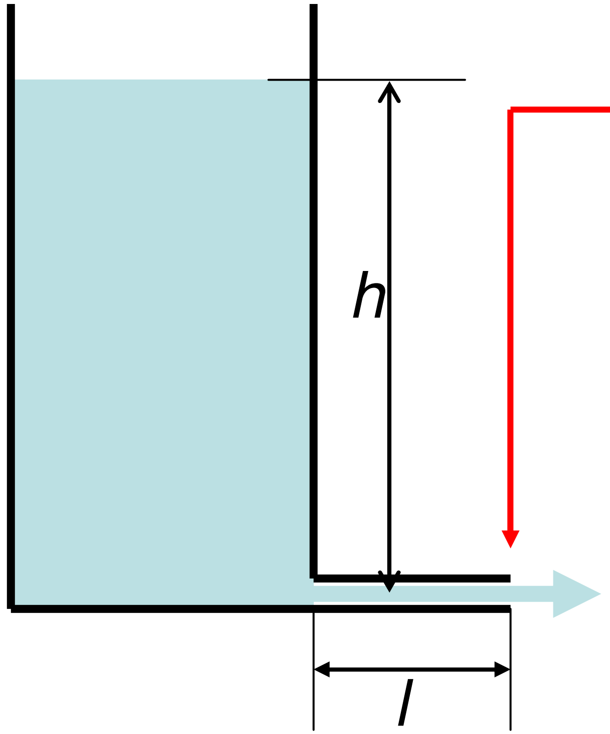
$$m \cdot a = V \cdot \rho \cdot \dot{v}$$

$$= A_p l \rho \cdot \dot{v} = -A_p \cdot \Delta p$$

$$\Rightarrow \quad -l\rho \cdot \frac{dv}{dt} = \Delta p$$

$$I_V = A_p \cdot v$$

625 Theorie



Induktivität

$$m \cdot a = V \cdot \rho \cdot \dot{v}$$

$$= A_p l \rho \cdot \dot{v} = -A_p \cdot \Delta p$$

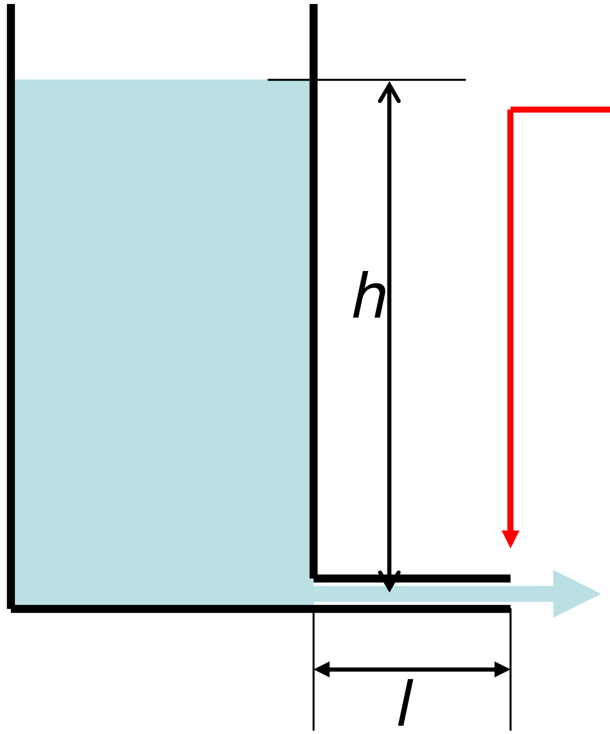
$$\Rightarrow \quad -l\rho \cdot \frac{dv}{dt} = \Delta p$$

$$I_V = A_p \cdot v$$

$$\Rightarrow \quad \frac{dI_V}{dt} = A_p \cdot \frac{dv}{dt}$$

625 Theorie

Induktivität



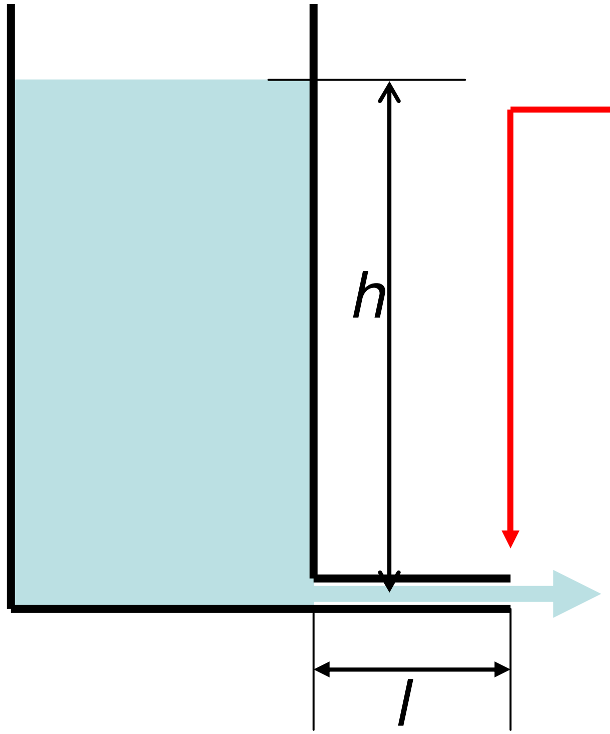
$$-l\rho \cdot \frac{dv}{dt} = \Delta p$$

$$\frac{dI_V}{dt} = A_p \cdot \frac{dv}{dt}$$

$$\Rightarrow \quad \Delta p = -\frac{l \cdot \rho}{A_p} \cdot \frac{dI_V}{dt}$$

625 Theorie

Induktivität



$$-l\rho \cdot \frac{dv}{dt} = \Delta p$$

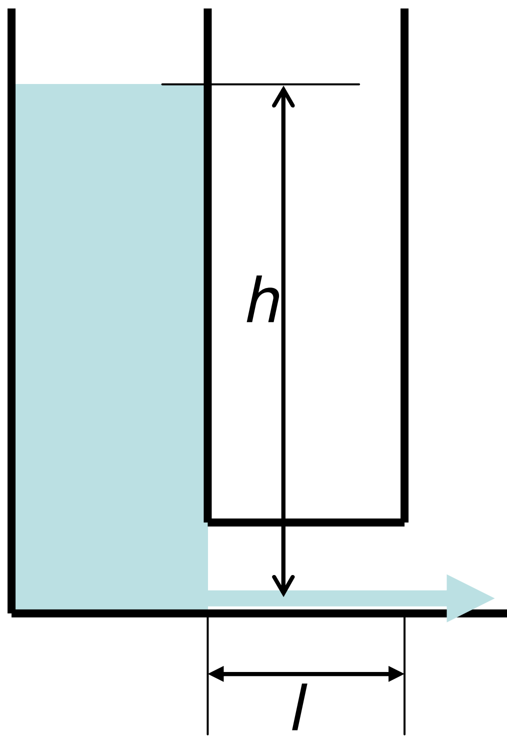
$$\frac{dI_V}{dt} = A_p \cdot \frac{dv}{dt}$$

$$\Rightarrow \quad \Delta p = -\frac{l \cdot \rho}{A_p} \cdot \frac{dI_V}{dt}$$

$$= -L \cdot \frac{dI_V}{dt}$$

625 Theorie

Kapazität und Induktivität



$$\frac{dV}{dt} = -\frac{1}{R}\cdot\Delta p - \frac{L}{R}\cdot\frac{dI_V}{dt}$$

$$C\cdot\Delta p = V$$

$$\boxed{\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{1}{LC}\cdot V = 0}$$

626 Schallwellen in Gasen und Flüssigkeiten, Dopplereffekt

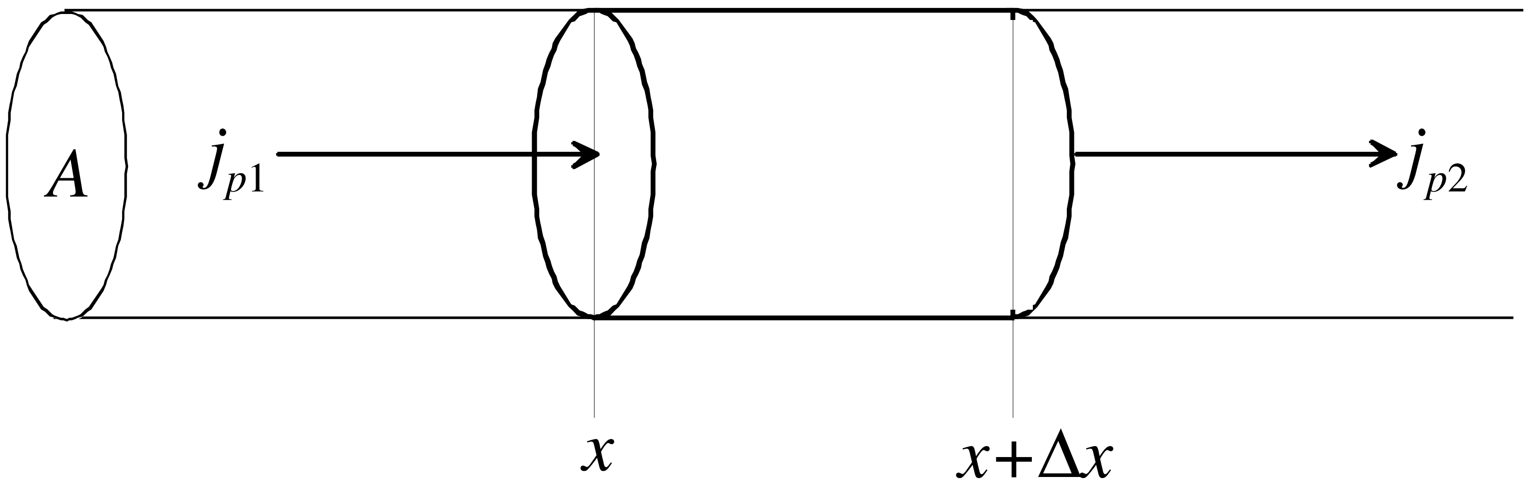
626 Ziele

- Wellengeschwindigkeit in Gasen und Flüssigkeiten berechnen können
- Dopplereffekt erklären und bei bewegten Quellen Frequenzverschiebung berechnen können

626 Theorie

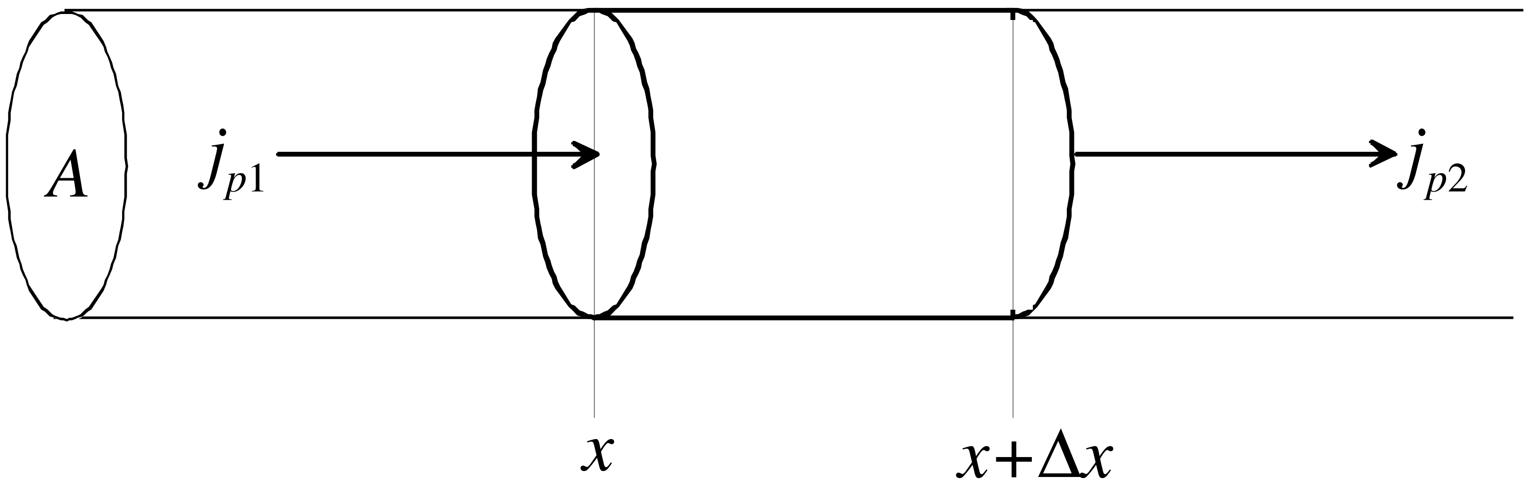
Longitudinale Verschiebung
bzw.Druckaufbau

→ Longitudinalwellen



626 Theorie

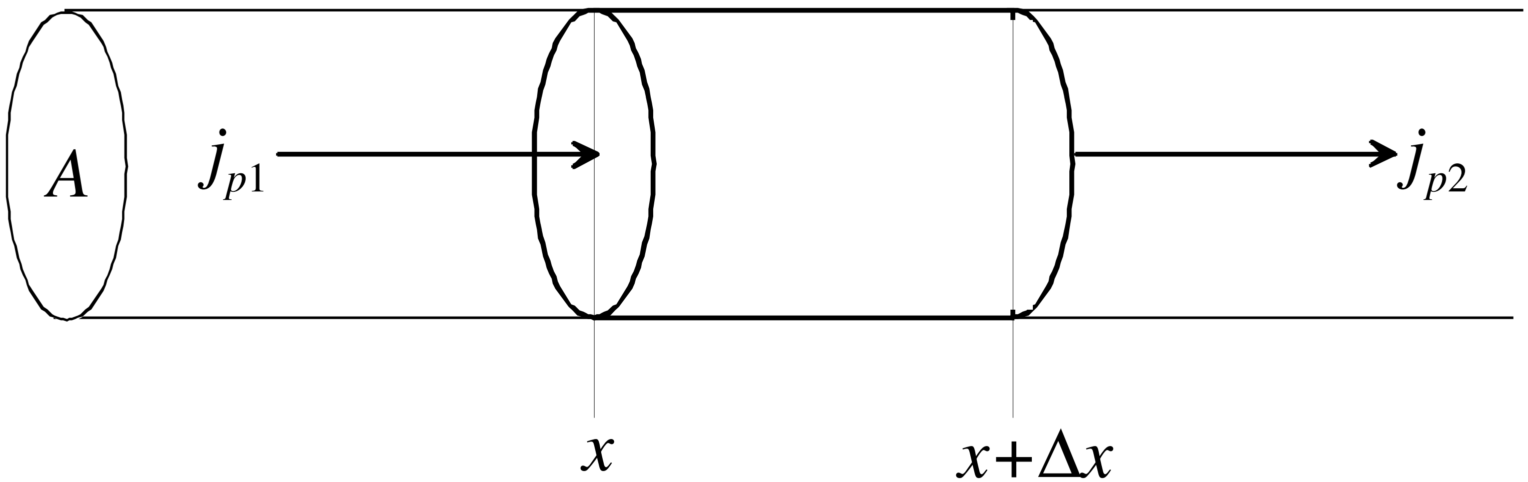
$$ma = \sum_i F_i = m \cdot \frac{dv}{dt} = F_1 + F_2$$



626 Theorie

$$ma = \sum_i F_i = m \cdot \frac{dv}{dt} = F_1 + F_2$$

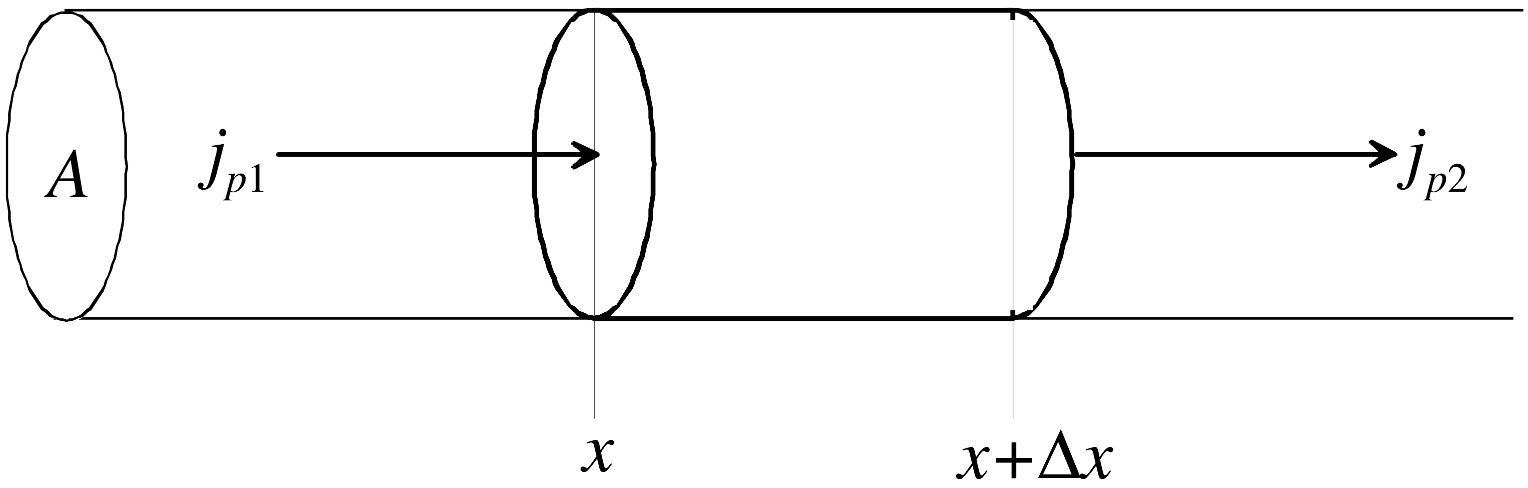
$$j_p = \pm \frac{F}{A} \longrightarrow m \cdot \frac{dv}{dt} = (j_{p1} - j_{p2}) \cdot A$$



626 Theorie

$$ma = \sum_i F_i = m \cdot \frac{dv}{dt} = F_1 + F_2$$

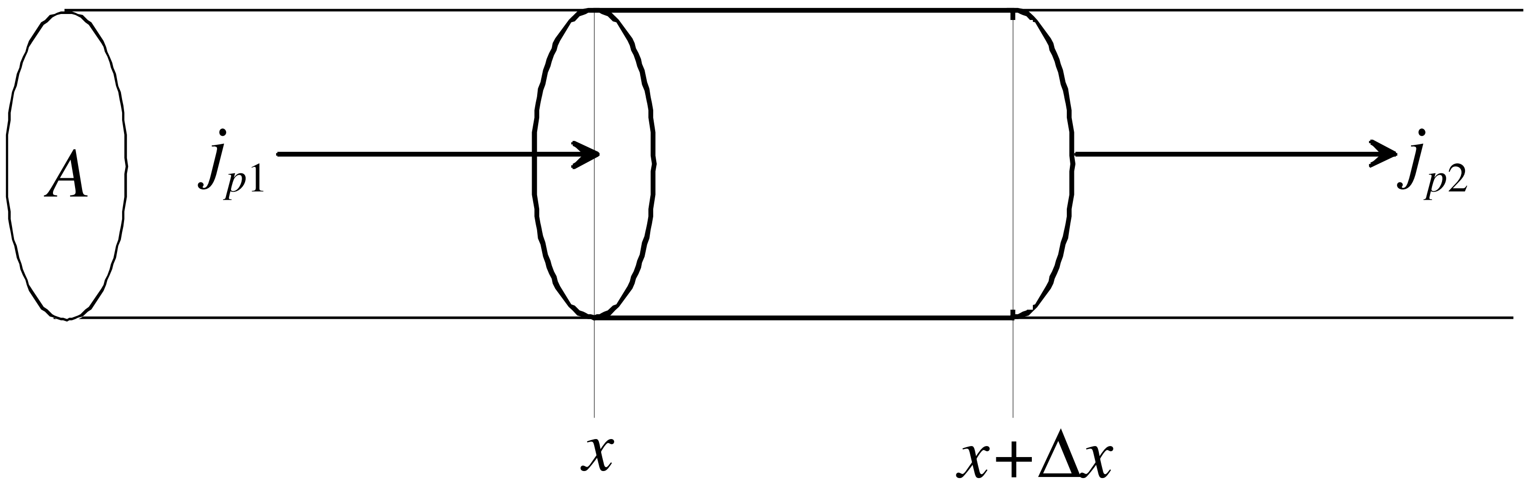
$$m \cdot \frac{dv}{dt} = (j_{p1} - j_{p2}) \cdot A = -(j_{p2} - j_{p1}) \cdot A = -A \cdot \Delta j_p$$



626 Theorie

$$m \cdot \frac{dv}{dt} = (j_{p1} - j_{p2}) \cdot A = -(j_{p2} - j_{p1}) \cdot A = -A \cdot \Delta j_p$$

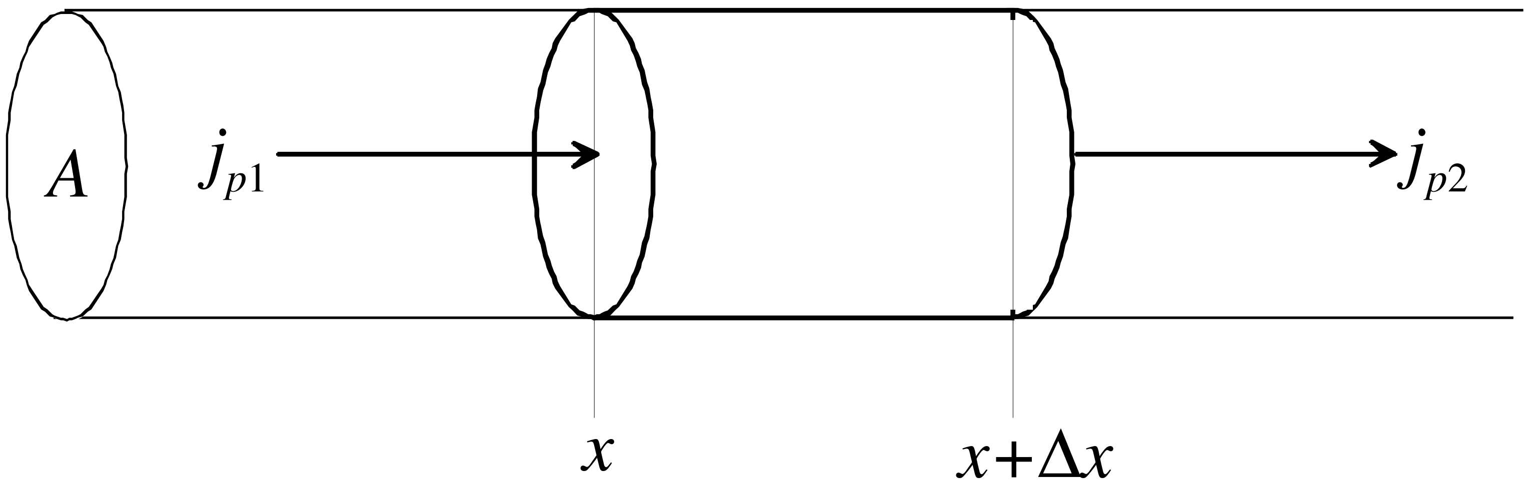
$$\frac{m}{\Delta x} \cdot \frac{dv}{dt} = C^* \cdot \frac{dv}{dt} = -A \cdot \frac{\Delta j_p}{\Delta x}$$



626 Theorie

$$\frac{m}{\Delta x} \cdot \frac{dv}{dt} = C^* \cdot \frac{dv}{dt} = -A \cdot \frac{\Delta j_p}{\Delta x}$$

$$C^* \cdot \frac{\partial v}{\partial t} = -A \cdot \frac{\partial j_p}{\partial x}$$



626 Theorie

$$C^* \cdot \frac{\partial v}{\partial t} = -A \cdot \frac{\partial j_p}{\partial x}$$

$$\left.\begin{array}{l} \Delta v \propto A \cdot \left(\dfrac{dj_p}{dt} \right) \\ \Delta v \propto \Delta x \\ \Delta v \propto L^* \end{array}\right\} \Delta v = -L^* \cdot A \cdot \frac{dj_p}{dt} \cdot \Delta x$$

626 Theorie

$$C^* \cdot \frac{\partial v}{\partial t} = -A \cdot \frac{\partial j_p}{\partial x}$$

$$\left.\begin{array}{l} \Delta v \propto A \cdot \left(\dfrac{dj_p}{dt}\right) \\ \Delta v \propto \Delta x \\ \Delta v \propto L^* \end{array}\right\} \Delta v = -L^* \cdot A \cdot \frac{dj_p}{dt} \cdot \Delta x$$

$$\Rightarrow \quad \frac{\partial v}{\partial x} = -L^* \cdot A \cdot \frac{\partial j_p}{\partial t}$$

626 Theorie

$$C^* \cdot \frac{\partial v}{\partial t} = -A \cdot \frac{\partial j_p}{\partial x}$$

$$\frac{\partial v}{\partial x} = -L^* \cdot A \cdot \frac{\partial j_p}{\partial t} \quad \Rightarrow \quad \partial j_p = -\frac{1}{L^* A} \cdot \frac{\partial v}{\partial x} \cdot \partial t$$

626 Theorie

$$\frac{\partial v}{\partial x} = -L^{*} \cdot A \cdot \frac{\partial j_{p}}{\partial t} \quad \Rightarrow \quad \partial j_{p} = -\frac{1}{L^{*} A} \cdot \frac{\partial v}{\partial x} \cdot \partial t$$

$$\frac{\partial v}{\partial t} = \frac{A}{L^{*} C^{*} A} \cdot \frac{1}{\partial x}\left[\frac{\partial v}{\partial x}\right] \cdot \partial t$$

$$\Big\downarrow \frac{\partial}{\partial t}$$

$$\Rightarrow \frac{\partial^{2} v}{\partial t^{2}} = \frac{1}{L^{*} C^{*}} \cdot \frac{\partial^{2} v}{\partial x^{2}}$$

626 Theorie

Ausbreitungsgeschwindigkeit
einer Longitudinalwellen

$$\frac{\partial^2 v}{\partial t^2} = \frac{1}{L^* C^*} \cdot \frac{\partial^2 v}{\partial x^2}$$

$$c = \frac{1}{\sqrt{L^* C^*}}$$

626 Theorie

$$\frac{\partial^2 v}{\partial t^2} = \frac{1}{L^* C^*} \cdot \frac{\partial^2 v}{\partial x^2}$$

$$c = \frac{1}{\sqrt{L^* C^*}}$$

Ausbreitungsgeschwindigkeit einer Longitudinalwellen

speziell Flüssigkeiten mit Kompressibilität κ:

$$\boxed{c = \frac{1}{\sqrt{\kappa \rho}}}$$

626 Theorie

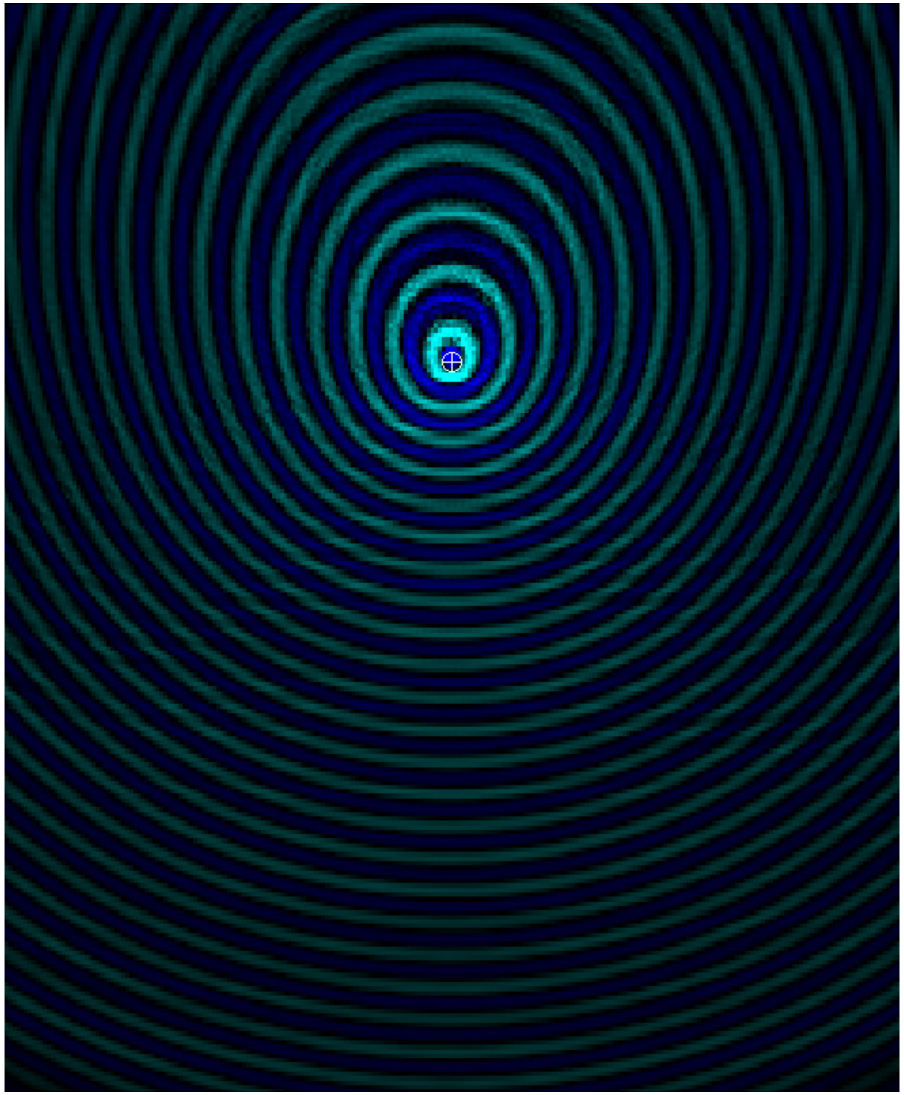
$$\frac{\partial^2 v}{\partial t^2} = \frac{1}{L^* C^*} \cdot \frac{\partial^2 v}{\partial x^2}$$

$$c = \frac{1}{\sqrt{L^* C^*}}$$

Ausbreitungsgeschwindigkeit einer Longitudinalwellen

speziell Gase mit Molmasse *M* bei einer Temperatur *T*:

$$\boxed{c = \sqrt{\chi \frac{RT}{M}}}$$



626 Theorie

Bewegte Schallquelle:

Frequenzverschiebung durch *“Stauchung“* oder *“Dehnung“* der Wellenfronten, Abhängig von der Geschwindigkeit des Beobachters v_B und derjenigen der Quelle v_Q

$$\boxed{\nu = \nu_0 \cdot \frac{c \pm v_B}{c \mp v_Q}}$$

627 Schallpegel

627 Ziele

- Schallintensität und Schallpegel definieren können
- Schallpegel aus Quellenleistung berechnen können (und Umkehrung)

627 Theorie

$$J = \frac{dP}{dA}$$

Schallintensität J

627 Theorie

$$J = \frac{dP}{dA}$$

Schallintensität *J*

$$J = \frac{J_{ref}}{r^2}$$

Schallintensität bei punktförmiger Quelle

627 Theorie

$$J = \frac{dP}{dA}$$

Schallintensität J

Schallintensität bei punktförmiger Quelle

$$J = \frac{J_{ref}}{r^2}$$

Schallpegel

$$L = 10 \cdot \log_{10}\left(\frac{J}{J_0}\right)$$

$$J_0 = 10^{-12} W / m^2$$